Logistic regression

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Prior Stata 11

When one might use logistic regression.

Some examples:

- One **binary** independent variable. (**one odds ratio**).
- Probabilities, odds and the logit function
- One **continuous** independent variable.
- One **categorical** independent variable. (The **Wald** test)
- One **binary** independent variable and **continuous** independent variable no interaction.
- One **binary** independent variable and **continuous** independent variable with interaction.

Watch out for **'small' reference** groups

The **likelihood ratio test**: comparing two nested models.

The logistic regression model in general

- The model and the **assumptions**.
- The **data** and the assumption of **independence**.
- **Estimation** and **inference**

Logistic regression models: Introduction

A logistic regression is a **possible** model if the **dependent** variable (the response) is **dichotomous** dead/alive obese/not obese etc.

Contrary to what many believe there are **no assumptions** about the **independent** variables.
They can be categorical or continuous.

When working with binary response it is **custom** to **code** the "**positive**" event (eg. dead) as **1** and a "**negative**" event (alive) as **0**.

A logistic regression models the **probability** of a "positive event" via odds.

And the associations via **odds ratio**.

If the **event is rare** then **odds ratios** estimate the **relative risk**.

Logistic regression models: Introduction

A logistic regression can also be used to estimate the odds ratios in a **unmatched case-control** study.

For such data the **constant** terms have **no meaning**.

And the odds ratios comparable odds ratio from a **follow-up study**.

Many **other epidemiological design** are analyzed by logistic regression models.

Estimating one odds ratio using logistic regresion

We are now considering a larger part of the Frammingham data set, consisting of 4690 person with **known BMI** at the start.

We will focus on the risk obesity (BMI≥30 kg/m^2) .

Out of the 4690 persons 601 = 12.8% were *obese*.

Divided into gender

	Obese	Not-Obese
Women	375 (14.2%)	2268
Men	226 (11.0%)	1821

We see a higher prevalence among women: OR: 1.33 (1.12;1.59).

That is **the odds** of being obese is between 12 and 59 percent higher for women.(χ^2=10.2 p-value=0.001)

Finding an odds ratio using logistic regresion

The odds ratio is defined as: $OR = \dfrac{odds_{Women}}{odds_{Men}}$

So applying the logarithm we get:

$$\ln(OR) = \ln\left(\frac{odds_{Women}}{odds_{Men}}\right) = \ln(odds_{Women}) - \ln(odds_{Men})$$

And rearranging terms :

$$\ln(odds_{Women}) = \ln(odds_{Men}) + \ln(OR)$$

That is the log-odds obesity for the women can be written as the sum of two terms:

- The log-odds in **reference** group (men)
- The log of the odds ratio

Finding an odds ratio using logistic regresion

$$\ln(odds_{Women}) = \ln(odds_{Men}) + \ln(OR)$$

If we again let *women* be a indicator/dummy variable, then we can consider the model:

$$\ln(odds) = \beta_0 + \beta_1 \cdot woman$$

For **men** we get: $\ln(odds) = \beta_0$

And for **women**: $\ln(odds) = \beta_0 + \beta_1$

Comparing with the equation on top we get:

$$\beta_0 = \ln(odds_{Men})$$

and

$$\beta_1 = \ln(OR)$$

Finding an odds ratio using logistic regresion

$$\ln(odds) = \underbrace{\beta_0}_{\ln(odds_{Men})} + \underbrace{\beta_1}_{\ln(OR)} \cdot woman$$

Or to be more precise: $\beta_1 = \ln(OR_{Women\,vs\,Men})$

So, if we can fit the model above to the data, then we can get an estimate of the $\log(OR)$ and hence of OR!

Probabilities and odds

If p denote the probability of an event (the **risk**, the **prevalence** proportion, or **cumulated incidence** proportion) then the odds is given by :

$$odds = \frac{p}{1-p}$$

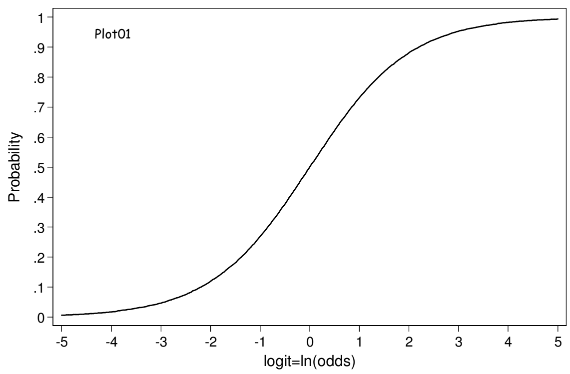
Note: $odds=1 \Leftrightarrow p=0.5 \Leftrightarrow \ln(odds)=0$

$$\ln(odds) = \ln\left(\frac{p}{1-p}\right)$$

In mathematics the last function of p is called the "logit" function.

$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right)$$

Probabilities and odds



Probabilities and odds

$$\boxed{\ln(odds) = \beta_0 + \beta_1 \cdot woman}$$

So modelling the **log-odds** is the same as modelling $\text{logit}(p)$ and model from before could be written.

$$\boxed{\text{logit}(p) = \beta_0 + \beta_1 \cdot woman}$$

Going from odds to probabilities: $p = \dfrac{odds}{1+odds}$

The model on **probability scale** is :

$$\boxed{p = \frac{\exp(\beta_0 + \beta_1 \cdot woman)}{1+\exp(\beta_0 + \beta_1 \cdot woman)} = INVLOGIT(\beta_0 + \beta_1 \cdot woman)}$$

Finding an odds ratio using logistic regresion

$$\text{logit}(p) = \ln(odds) = \beta_0 + \beta_1 \cdot woman$$

Back to finding the estimates.

In Stata:

```
char sex[omit]1
xi: logit obese i.sex
```

```
i.sex             _Isex_1-2           (naturally coded; _Isex_1 omitted)
Iteration 0:   log likelihood = -1795.5437
Iteration 3:   log likelihood = -1790.3703
Logit estimates                                   Number of obs   =       4690
                                                  LR chi2(1)      =      10.35
                                                  Prob > chi2     =     0.0013
Log likelihood = -1790.3703                       Pseudo R2       =     0.0029
------------------------------------------------------------------------------
   obese |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------+--------------------------------------------------------------------
 _Isex_2 |   .2868784   .0898972     3.19   0.001     .1106831    .4630738
   _cons |  -2.086606    .070526   -29.59   0.000    -2.224835   -1.948378
------------------------------------------------------------------------------
```

Finding an odds ratio using logistic regression

$$\text{logit}(p) = \ln(odds) = \beta_0 + \beta_1 \cdot woman$$

$\hat{\beta}_1 = \ln\left(\widehat{OR}\right)$ 95% CI for $\ln(OR)$

```
--------------------------------------------------------------------------
 obese |    Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------+------------------------------------------------------------------
_Isex_2 | .2868784   .0898972    3.19   0.001    .1106831     .4630738
  _cons | -2.086606   .070526  -29.59   0.000   -2.224835    -1.948378
--------------------------------------------------------------------------
```

$\widehat{OR} = \exp(0.2868784) = 1.33$ 95% CI: $(1.12;1.59)$.

Test for the hypothesis : $\ln(OR)=0 \Leftrightarrow OR=1$

Odds in **reference** group (men) = exp(-2.086606)=0.1241

95% CI :(0.1081;0.1425).

Prevalence among men: 0.1104 (0.0975;0.1247).

Finding an odds ratio using logistic regression

$$\text{logit}(p) = \ln(odds) = \beta_0 + \beta_1 \cdot woman$$

An easier way to obtain the odds ratio.

xi: *logit* obese *i*.sex **,or**

```
i.sex            _Isex_1-2           (naturally coded; _Isex_1 omitted)
Iteration 0:    log likelihood = -1795.5437
Iteration 3:    log likelihood = -1790.3703
Logit estimates
                                                   Number of obs   =       4690
                                                   LR chi2(1)      =      10.35
                                                   Prob > chi2     =     0.0013
Log likelihood = -1790.3703                        Pseudo R2       =     0.0029
------------------------------------------------------------------------------
  obese | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------+---------------------------------------------------------------------
_Isex_2 |   1.332262   .1197667    3.19   0.001     1.117041    1.588951
------------------------------------------------------------------------------
```

Note, we cannot find any information about the risk in the reference group , i.e. the odds and prevalence among men!

The obesity and age: version 1

In the previous section we saw that the prevalence of obesity was different between men and women.

Is it also associated with age?

The simplest model **on the logit scale** would be:

$$\text{logit}(p) = \ln(odds) = \beta_0 + \beta_1 \cdot age$$

That is a linear relation on the log-odds scale.

As we have seen before using *age* implies that β_0 references to a newborn (*age*=0).

So we will chose *age*=45 reference instead:

$$\text{logit}(p) = \ln(odds) = \beta_0 + \beta_1 \cdot (age - 45)$$

The obesity and age: version 1

$$\text{logit}(p) = \ln(odds) = \beta_0 + \beta_1 \cdot (age - 45)$$

The interpretation of the parameters:

β_0 : the **log odds** for 45 year old person.

β_1 : the **log odds ratio**, when comparing two persons who differ 1 year in age.

$\exp(\beta_1)$: the **odds ratio**, when comparing two persons who differ 1 year in age.

Note, that this odds ratio is **assumed** to be the same no matter what age the two persons have, as long as they differ by one year!

The log odds ratio is **proportional** to the age differences,

e.g. OR increases **exponentially** with the age differences.

The obesity and age: version 1

$$\text{logit}(p) = \ln(odds) = \beta_0 + \beta_1 \cdot (age - 45)$$

Obtaining the estimates in Stata:

```
generate age45=age-45
logit obese age45
```

```
------------------------------------------------------------------------------
obese |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
------+-----------------------------------------------------------------------
age45 |   .0348023   .0051296     6.78   0.000     .0247484    .0448561
_cons |  -1.985922   .0463594   -42.84   0.000    -2.076785   -1.895059
------------------------------------------------------------------------------
```

Test for no association with *age*

```
logit obese age45,OR
```

```
------------------------------------------------------------------------------
obese | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
------+-----------------------------------------------------------------------
age45 |   1.035415   .0053113     6.78   0.000     1.025057    1.045877
------------------------------------------------------------------------------
```


The obesity and age: version 1

$$\text{logit}(p) = \ln(odds) = \beta_0 + \beta_1 \cdot (age - 45)$$

Estimate: β_0 : −1.985 (−2.0767;−1.8951)

The **odds** for obesity for **among 45 year** old:

0.1373 (0.1253;0.1503)

The **prevalence** of obesity for **among 45 year** old:

0.1207 (0.1114;0.1307)

$$odds = \exp(\log(odds)) \qquad \text{Prob} = \frac{odds}{1 + odds}$$

The obesity and age: version 1

$$\text{logit}(p) = \ln(odds) = \beta_0 + \beta_1 \cdot (age - 45)$$

Estimates: β_1 : 0.0348 (0.0247;0.0449)

The **odds ratio** for being obese is 1.0354 (1.0251;1.0459) when comparing the old person to the young person, if they differ with **one year in age**.

If they differ with **4.5 years** then the odds ratio is

$1.0354^{4.5}$ $(1.0251^{4.5};1.0459^{4.5})$= 1.17 (1.12;1.22)

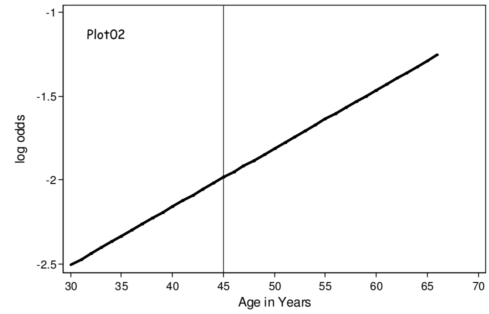
In Stata: `lincom age45*4.5,OR`

```
 ( 1)  4.5 age45 = 0
```

obese	Odds Ratio	~~Std. Err.~~	z	P>|z|	[95% Conf. Interval]	
(1)	1.16954	~~.0269968~~	6.78	0.000	1.117806	1.223668

The obesity and age: version 1

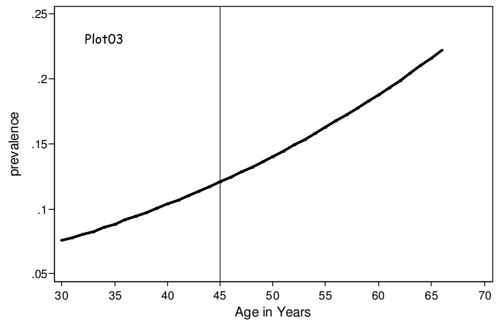
Estimated relationship: $\ln(odds) = -1.986 + 0.0348 \cdot (age - 45)$



The obesity and age: version 1

Estimated relationship:

$$prevalence = \frac{\exp(-1.986 + 0.0348 \cdot (age - 45))}{1 + \exp(-1.986 + 0.0348 \cdot (age - 45))}$$



The obesity and age: version 2

$$\ln(odds) = \beta_0 + \beta_1 \cdot (age - 45)$$

This model assumes that one year of age difference is associated with the same odds ratio irrespectively of the age.

An other way to model the prevalence could be to assume a step function that is to categorize age.

We will here look at age divided in seven five-years groups:

`egen agegrp7=cut(age), at(0,35,40,45,50,55,60,120) label`

With this command the **youngest** age group will be number 0 the **second youngest**: 1 and the **oldest**: 6

The obesity and age: version 2

`table agegrp7 ,c(min age max age count obese sum obese) row`

agegrp7	min(age)	max(age)	N(obese)	sum(obese)
0-	30	34	352	23
35-	35	39	973	105
40-	40	44	885	93
45-	45	49	799	95
50-	50	54	733	115
55-	55	59	613	95
60-	60	66	335	75
Total	30	66	4,690	601

A model that have different odds in each age group :

$$\ln(odds) = \alpha_0 + \sum_{i=1}^{6} \alpha_i \cdot agei$$

Where $agei$ is an indicator for being in the ith age group

The obesity and age: version 2

$$\ln(odds) = \alpha_0 + \sum_{i=1}^{6} \alpha_i \cdot agei$$

The interpretation of the parameters:

α_0 : the **log odds** in **reference** group=the youngest.

α_i : the **log odds ratio**, when comparing one person in age group i with one in the reference group=the youngest.

```
char agegrp7[omit]0
xi: logit obese i.agegrp7
```

Not all output

obese	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
_Iagegrp7_1	.54833	.23915	2.29	0.022	.079603	1.017061
_Iagegrp7_2	.51860	.24193	2.14	0.032	.0444155	.992787
_Iagegrp7_3	.65766	.24179	2.72	0.007	.1837537	1.13157
_Iagegrp7_4	.97900	.23839	4.11	0.000	.5117642	1.44625
_Iagegrp7_5	.96446	.24284	3.97	0.000	.4884941	1.440436
_Iagegrp7_6	1.41737	.25238	5.62	0.000	.9227081	1.912032
_cons	-2.66056	.21567	-12.34	0.000	-3.083288	-2.237839

The obesity and age: version 2

$$\ln(odds) = \alpha_0 + \sum_{i=1}^{6} \beta_i \cdot agei$$

xi: logit obese i.agegrp7,or **Not all output**

obese	Odds Ratio	Std. Err	z	P>|z|	[95% Conf.	Interval]
_Iagegrp7_1	1.730365	~~.4138201~~	2.29	0.022	1.082857	2.765057
_Iagegrp7_2	1.679677	~~.4063746~~	2.14	0.032	1.045417	2.698747
_Iagegrp7_3	1.930274	~~.4667295~~	2.72	0.007	1.20172	3.100522
_Iagegrp7_4	2.661812	~~.6345592~~	4.11	0.000	1.668232	4.247159
_Iagegrp7_5	2.623384	~~.6370806~~	3.97	0.000	1.62986	4.222538
_Iagegrp7_6	4.126254	~~1.041397~~	5.62	0.000	2.516095	6.766825

The OR between the **second oldest** and the **youngest**:

2.62 (1.63;4.22)

Between a 63 and 322 percent **increase** in odds.

Small prevalence: 63 and 322 percent **increase** in prevalence.

A statistical significant difference in prevalence!

The obesity and age: version 2

$$\ln(odds) = \alpha_0 + \sum_{i=1}^{6} \alpha_i \cdot agei$$

The output contains **six tests** of no difference in risk – comparing each of the six groups with the **reference** (the youngest) group.

The command: *testparm* _Iagegrp*
will give a **"Wald test"** of no difference between the **seven** groups .

```
 ( 1)  _Iagegrp7_1 = 0
 ( 2)  _Iagegrp7_2 = 0
 ( 3)  _Iagegrp7_3 = 0
 ( 4)  _Iagegrp7_4 = 0
 ( 5)  _Iagegrp7_5 = 0
 ( 6)  _Iagegrp7_6 = 0
           chi2(  6) =    55.26
         Prob > chi2 =    0.0000
```

Highly significant differences

The obesity and age: version 2

Using the age group 45-49 as **reference**

char agegrp7[omit]3

xi: logit obese i.agegrp7,or **Not all output**

obese	Odds Ratio	Std. Err.	z	P>|z|	[95% Conf.	Interval]
_Iagegrp7_0	.518061	~~.1252643~~	-2.72	0.007	.3225264	.8321407
_Iagegrp7_1	.896434	~~.1348312~~	-0.73	0.467	.6675609	1.203778
_Iagegrp7_2	.870175	~~.1347005~~	-0.90	0.369	.6424561	1.17861
_Iagegrp7_4	1.378981	~~.2057436~~	2.15	0.031	1.029341	1.847385
_Iagegrp7_5	1.359073	~~.2123097~~	1.96	0.050	1.000625	1.845927
_Iagegrp7_6	2.137652	~~.3648206~~	4.45	0.000	1.529915	2.986803

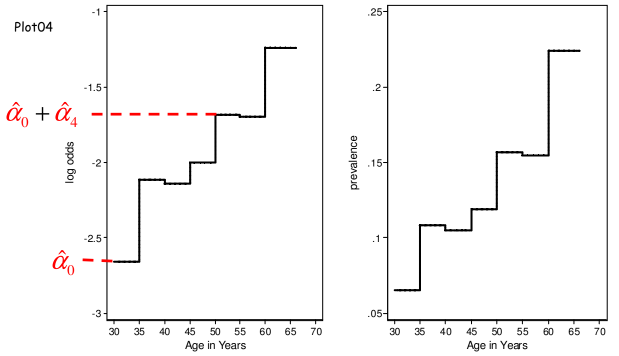
The OR between the **second oldest** and the **45-49 old**:

1.36 (1.00;1.85)

Between a **no** and 85 percent **increase** in (odds) prevalence.

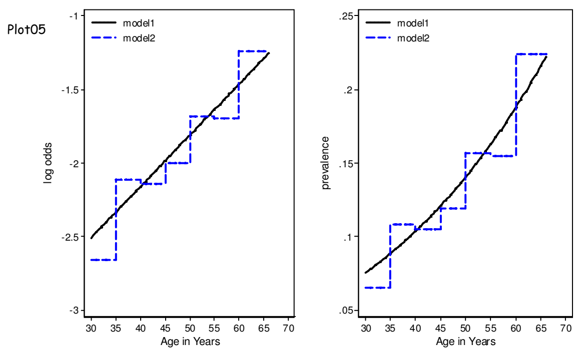
A borderline significant different in prevalence!

The obesity and age: version 2



Estimated relationship

The obesity and age: version 1 and 2



The obesity, sex and age: version 1

The first analysis only looked at sex and the second only at age.

Let us try to look at those two at the same time

The simplest model **on the logit scale** would be:

$$\ln(odds) = \beta_0 + \beta_1 \cdot woman + \beta_2 \cdot (age - 45)$$

This is based on three **assumptions**:

Additivity on **logit scale**: The contribution from sex and age are **added**.

Proportionalty on **logit scale**: The contribution from age is **proportional** to it is value.

No effectmodification on **logit scale**: The contribution from one independent variable **is the same** whatever the value is for the other.

The obesity, sex and age : version 1

$$\ln(odds) = \beta_0 + \beta_1 \cdot woman + \beta_2 \cdot (age - 45)$$

The interpretation of the parameters:

β_0 : the **log odds** for 45 year old **man**.

β_1 : the **log odds ratio**, when comparing a woman to a man of the same age.

β_2 : the **log odds ratio**, when comparing two persons of the same sex, where the first is one year older than the other.

β_2 *Δage: the **log odds ratio**, when comparing two persons of the same sex, where the first is Δage years older than the other.

The obesity, sex and age : version 1

$$\ln(odds) = \beta_0 + \beta_1 \cdot woman + \beta_2 \cdot (age - 45)$$

Obtaining the estimates in Stata:

xi:logit obese *i.*sex age45

```
i.sex            _Isex_1-2          (naturally coded; _Isex_1 omitted)
Iteration 0:   log likelihood = -1795.5437
Iteration 3:   log likelihood = -1767.7019
Logit estimates                                   Number of obs   =       4690
                                                  LR chi2(2)      =      55.68
                                                  Prob > chi2     =     0.0000
Log likelihood = -1767.7019                       Pseudo R2       =     0.0155
------------------------------------------------------------------------------
   obese |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------+--------------------------------------------------------------------
 _Isex_2 |   .2743977   .0903385     3.04   0.002     .0973375     .451458
   age45 |   .0344723   .0051354     6.71   0.000     .0244072    .0445374
   _cons |  -2.147056   .0721981   -29.74   0.000    -2.288561    -2.00555
------------------------------------------------------------------------------
```

Tests: No association with sex No association with age

Prevalence is 50% among 45 year old men

The obesity, sex and age : version 1

$$\ln(odds) = \beta_0 + \beta_1 \cdot woman + \beta_2 \cdot (age - 45)$$

```
xi:logit obese i.sex age45, or
   obese | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------+--------------------------------------------------------------------
 _Isex_2 |   1.315738   .1188618     3.04   0.002     1.102232      1.5706
   age45 |   1.035073   .0053155     6.71   0.000     1.024707    1.045544
------------------------------------------------------------------------------
```

OR for **women** compared to men "adjusted for age" :
$1.32\ (1.10;1.57)$

The **unadjusted** was $1.33\ (1.12;1.59)$.

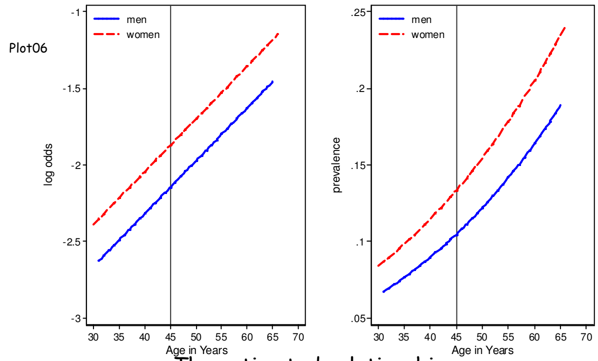
OR for **one year age** difference "adjusted for sex" :
$1.04\ (1.02;1.05)$

The **unadjusted** was $1.04\ (1.03;1.05)$

Not much has changed!

The obesity, sex and age : version 1

$$\ln(odds) = \beta_0 + \beta_1 \cdot woman + \beta_2 \cdot (age - 45)$$



The estimated relationship

The obesity, sex and age: version 2

A more complicated model **on the logit scale** would be:

men: $\ln(odds) = \alpha_0 + \alpha_1 \cdot (age - 45)$

women: $\ln(odds) = \gamma_0 + \gamma_1 \cdot (age - 45)$

This is based on one **assumptions**:

Proportionalty on **logit scale**: The contribution age is **proportional** to it is value.

It can be written in just one formula (with interaction):

$$\ln(odds) = \beta_0 + \beta_1 \cdot woman + \beta_2 \cdot (age - 45) + \beta_3 \cdot woman \cdot (age - 45)$$

Where: $\alpha_0 = \beta_0$ $\quad \alpha_1 = \beta_2$

$\gamma_0 = \beta_0 + \beta_1$ $\quad \gamma_1 = \beta_2 + \beta_3$

That is: $\beta_1 = \gamma_0 - \alpha_0$ $\quad \beta_3 = \gamma_1 - \alpha_1$

The obesity, sex and age: version 2

$$\ln(odds) = \beta_0 + \beta_1 \cdot woman + \beta_2 \cdot (age - 45) + \beta_3 \cdot woman \cdot (age - 45)$$

Estimates log odds:

```
xi: logit obese i.sex*age45
------------------------------------------------------------------------------
       obese |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     _Isex_2 |    .116797    .095034     1.23   0.219    -.069467     .303061
       age45 |  -.0056849    .008372    -0.68   0.497    -.022095     .010725
_IsexXage4~2 |    .065803     .01074     6.13   0.000     .044747    .0868588
       _cons |  -2.083041    .070643   -29.49   0.000    -2.22149   -1.944583
------------------------------------------------------------------------------
```

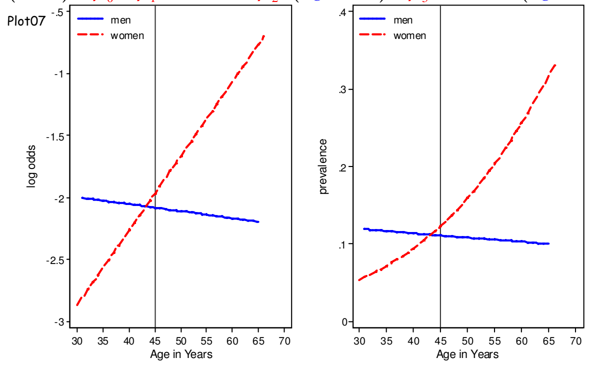
Men Difference between women and men

Estimates odds ratios:

```
       obese | Odds Ratio   Std. Err      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     _Isex_2 |   1.123891     .10680     1.23   0.219     .9328908    1.353997
       age45 |    .994331     .00832    -0.68   0.497     .978147    1.010783
_IsexXage4~2 |   1.068016     .01147     6.13   0.000    1.045763    1.090743
------------------------------------------------------------------------------
```

The obesity, sex and age: version 2

$$\ln(odds) = \beta_0 + \beta_1 \cdot woman + \beta_2 \cdot (age - 45) + \beta_3 \cdot woman \cdot (age - 45)$$



The estimated relationship

A small case-control example

```
tabodds cancer age

  age  |     cases    controls      odds     [95% Conf. Interval]
-------+----------------------------------------------------------
25-34  |         2        116     0.01724      0.00426    0.06976
35-44  |         9        190     0.04737      0.02427    0.09244
45-54  |        46        167     0.27545      0.19875    0.38175
55-64  |        76        166     0.45783      0.34899    0.60061
65-74  |        55        106     0.51887      0.37463    0.71864
 >=75  |        13         31     0.41935      0.21944    0.80138
```

Few events in reference group= wide CI's

```
tabodds cancer age, or

  age  | Odds Ratio      chi2       P>chi2     [95% Conf. Interval]
-------+-------------------------------------------------------------
25-34  |   1.000000        .           .            .          .
35-44  |   2.747368      1.76       0.1843     0.579474  13.025660
45-54  |  15.976048     24.18       0.0000     3.588609  71.123412
55-64  |  26.554217     41.14       0.0000     5.834718 120.850133
65-74  |  30.094340     43.99       0.0000     6.278745 144.243682
 >=75  |  24.322581     29.40       0.0000     4.402342 134.380270
```


A small case-control example

```
tabodds cancer age

  age  |     cases    controls      odds     [95% Conf. Interval]
-------+----------------------------------------------------------
25-34  |         2        116     0.01724      0.00426    0.06976
35-44  |         9        190     0.04737      0.02427    0.09244
45-54  |        46        167     0.27545      0.19875    0.38175
55-64  |        76        166     0.45783      0.34899    0.60061
65-74  |        55        106     0.51887      0.37463    0.71864
 >=75  |        13         31     0.41935      0.21944    0.80138
```

'Many' events in reference group= narrow CI's

```
tabodds cancer age, or base(3)

  age  | Odds Ratio      chi2       P>chi2     [95% Conf. Interval]
-------+-------------------------------------------------------------
25-34  |   0.062594     24.18       0.0000     0.014060   0.278660
35-44  |   0.171968     25.86       0.0000     0.079661   0.371235
45-54  |   1.000000        .           .            .          .
55-64  |   1.662127      5.54       0.0186     1.083844   2.548952
65-74  |   1.883716      7.32       0.0068     1.181689   3.002809
 >=75  |   1.522440      1.30       0.2546     0.734799   3.154365
```


A small case-control example

"Many" iterations

```
char age [omit]1
 xi:logit cancer i.smoker i.age,or
i.smoker          _Ismoker_0-1        (naturally coded; _Ismoker_0 omitted)
i.age             _Iage_1-6           (naturally coded; _Iage_1 omitted)
Iteration 0:   log likelihood = -496.55682
Iteration 1:   log likelihood = -437.55133
Iteration 2:   log likelihood = -429.86007
Iteration 3:   log likelihood = -428.99383
Iteration 4:   log likelihood = -428.94473
Iteration 5:   log likelihood = -428.94432
Iteration 6:   log likelihood = -428.94432
Logit estimates                                   Number of obs   =        977
                                                  LR chi2(6)      =     135.23
                                                  Prob > chi2     =     0.0000
Log likelihood = -428.94432                       Pseudo R2       =     0.1362
------------------------------------------------------------------------------
    cancer | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----------+------------------------------------------------------------------
_Ismoker_1 |    2.350    .4513038     4.45   0.000     1.613342    3.424472
   _Iage_2 |    2.832     2.24368     1.31   0.189     .5995103     13.3798
   _Iage_3 |    16.58    12.17378     3.82   0.000     3.932286    69.91422
   _Iage_4 |    27.89    20.32374     4.57   0.000     6.691356    116.3235
   _Iage_5 |    34.79    25.59029     4.83   0.000     8.231516    147.0764
   _Iage_6 |    27.71    21.89267     4.21   0.000     5.891878    130.3509
------------------------------------------------------------------------------
```


A small case-control example

```
char age [omit]3
xi:logit cancer i.smoker i.age,or
i.smoker          _Ismoker_0-1        (naturally coded; _Ismoker_0 omitted)
i.age             _Iage_1-6           (naturally coded; _Iage_3 omitted)
Iteration 0:   log likelihood = -496.55682
Iteration 1:   log likelihood = -437.55133
Iteration 2:   log likelihood = -429.86007
Iteration 3:   log likelihood = -428.99383
Iteration 4:   log likelihood = -428.94473
Iteration 5:   log likelihood = -428.94432
Logit estimates                                   Number of obs   =        977
                                                  LR chi2(6)      =     135.23
                                                  Prob > chi2     =     0.0000
Log likelihood = -428.94432                       Pseudo R2       =     0.1362
------------------------------------------------------------------------------
    cancer | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----------+------------------------------------------------------------------
_Ismoker_1 |   2.3504     .451303     4.45   0.000     1.613343    3.424469
   _Iage_1 |    .0603    .0442767    -3.83   0.000     .0143051    .2542718
   _Iage_2 |    .1708    .0652397    -4.63   0.000     .0807999    .3610977
   _Iage_4 |   1.6826    .3701188     2.37   0.018     1.093327     2.58953
   _Iage_5 |   2.0984    .5042862     3.08   0.002      1.31025    3.360918
   _Iage_6 |   1.6713    .6277714     1.37   0.171     .8005146    3.489699
------------------------------------------------------------------------------
```


Things to look out for in the output

In general:

Wide CI's or **large standard errors** in a logistic regression indicates that at least one group has **few events**!

Many iterations in a logistic regression indicates that some of the **parameters are hard to estimate**.

Comparing two models: the likelihood ratio test

Earlier we saw how one could use a **Wald** to test if several coefficients could be zero .

An other way to "compare" two models is by a **likelihood ratio test**.

In the logistic regression output from Stata we find a likelihood ratio test comparing the **fitted model** with the model with no dependent variables the **constant odds model**:

```
LR chi2(6)      =     135.23
Prob > chi2     =     0.0000
```

The conclusion: The model with smoker and age is **statistical significant** better, than a model assuming the same odds, risk for everybody.

Morten Frydenberg Linear and Logistic regression - Note 4 43

Comparing two models: the likelihood ratio test

One can compare two models with a likelihood ratio test if:

- The two models are fitted on exactly the **same data set**.
- The two models are **nested**, i.e. one can go from one model to the other by setting some coefficients to zero.

In Stata the test is found in this way:

```
xi:logit cancer i.smoker i.age
estimates store model1
xi:logit cancer i.smoker
estimates store model2
lrtest model1 model2
```

Output:

```
likelihood-ratio test                              LR chi2(5)  =    120.82
(Assumption: model2 nested in model1)              Prob > chi2 =    0.0000
```

i.age adds **statistical significant** information to the model only containing smoking!

Morten Frydenberg Linear and Logistic regression - Note 4 44

Logistic regression model in general

$$\ln(odds) = \beta_0 + \sum_{p=1}^{k} \beta_p \cdot x_p$$

This is based on three assumptions:

a. **Additivity on log-odds scale**: The contribution from each of the independent variables are **added**.

b. **Proportionalty**: The contribution from independent variables is **proportional** to it is value (with a factor β)

c. **No effectmodification**: The contribution from one independent variables **is the same** whatever the values are for the other.

Note **a.** can also be formulate as **multiplicativity** on **odds scale**

$$odds = odds_0 \cdot OR_1^{x_1} \cdot OR_2^{x_2} \cdots OR_k^{x_k}$$

Morten Frydenberg Linear and Logistic regression - Note 4 45

Logistic regression model in general

$$\ln(odds) = \beta_0 + \sum_{p=1}^{k} \beta_p \cdot x_p$$

If one consider two persons who differ with

Δx_1 in x_1 , Δx_2 in x_2 ... and Δx_k in x_k

then difference in the **log odds** is :

$$\sum_{p=1}^{k} \beta_p \cdot \Delta x_p$$

Again we see that the contribution for each of the explanatory variables:

are **added**,
are **proportional** to the difference
and **does not dependent** of the difference in the other

on the log odds scale.

Morten Frydenberg Linear and Logistic regression - Note 4 46

Logistic regression model in general

$$\ln(odds) = \beta_0 + \sum_{p=1}^{k} \beta_p \cdot x_p$$

If one consider two persons who differ with

Δx_1 in x_1 , Δx_2 in x_2 ... and Δx_k in x_k

then odds ratio : $OR = OR_1^{\Delta x_1} \cdot OR_2^{\Delta x_2} \cdots OR_k^{\Delta x_k}$

Note the model might also be formulated:

$$p = \Pr[Y=1] = \frac{\exp\left(\beta_0 + \sum_{p=1}^{k} \beta_p \cdot x_p\right)}{1+\exp\left(\beta_0 + \sum_{p=1}^{k} \beta_p \cdot x_p\right)}$$

Morten Frydenberg Linear and Logistic regression - Note 4 47

Logistic regression model in general

$$\ln(odds) = \beta_0 + \sum_{p=1}^{k} \beta_p \cdot x_p$$

The data: $Y = 1/0$ dichotomous dependent variable

x_1 , x_2 ... x_k independent/explanatory variables

Like in the normal regression models it is assumed that the Y's are **independent** given the explanatory variables.

This assumption can, in general, only be checked by **scrutinising** the design.

Look out for data sampled in **clusters**:

Patients within the **same GP**

Children within the **same family**

Twins.

Morten Frydenberg Linear and Logistic regression - Note 4 48

Logistic regression model in general

Estimation:

Excepting the two by two tables, there are **no closed form** for the estimates.

The **distribution** of the estimates **are not known**.

Estimates are found by the method of **maximum likelihood**.

Estimates are using **iterative methods**.

Standard errors, confidence intervals and all tests are based on **asymptotics**.

That is, all statistical **inference** are **approximate**.

The **more data** – the more events -the **better** the approximations.