Multiple linear regression 2
Stata 11

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Categorical variables in regression models.

The changing **reference** level

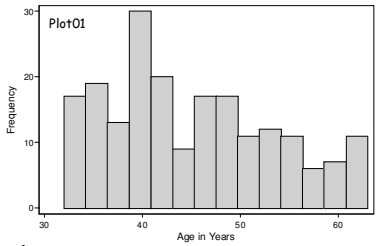
Interaction/effect modification

Interaction between a **categorical** and **continuous** variable

Interaction between two **categorical** variables

Categorical variable in regression models

The age distribution:



Let us divide age into **three** agegroups ,

0: $age \le 40$, 1: $40 < age \le 50$, 2: $50 < age$

and consider the new model

$$\ln(sbp) = \alpha_0 + \alpha_1 \cdot age1 + \alpha_2 \cdot age2 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

Categorical variable : age group 0 reference

$$\ln(sbp) = \alpha_0 + \alpha_1 \cdot age1 + \alpha_2 \cdot age2 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

$age1$ is one if a person is in age group **1** and zero otherwise
$age2$ is one if a person is in age group **2** and zero otherwise

The expected $\ln(sbp)$ in the three age groups will be:

$$age < 40: \quad \ln(sbp) = \alpha_0 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln(bmi/25)$$
$$40 \le age < 50: \quad \ln(sbp) = \alpha_0 + \alpha_1 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln(bmi/25)$$
$$50 \le age: \quad \ln(sbp) = \alpha_0 + \alpha_2 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln(bmi/25)$$

We see that α_1 is the adjusted difference in $\ln(sbp)$ when comparing a person in the **second group** with one in the **first group**.

And α_2 is the adjusted difference in $\ln(sbp)$ when comparing a person in the **third group** with one in the **first group**.

Categorical variable : age group 0 reference

$$\ln(sbp) = \alpha_0 + \alpha_1 \cdot age1 + \alpha_2 \cdot age2 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

Finally we see that α_0 is the expected $\ln(sbp)$ for a **man** in the **first** (reference) age group, with bmi=25.

In most programs the model is fitted by first generating the grouping variable and then making the regression telling the program which variables are categorical.

In Stata this done like this:

```
egen agegrp3=cut(age), at(0,40,50,120) label
  regress lnSBP woman i.agegrp3 lnBMI25
```

This is categorical

Categorical variable : age group 0 reference

$$\ln(sbp) = \alpha_0 + \alpha_1 \cdot age1 + \alpha_2 \cdot age2 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

OUTPUT:

```
      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  4,   195) =   11.20
       Model |  .980169926     4  .245042482           Prob > F      =  0.0000
    Residual |  4.26524771   195  .021873065           R-squared     =  0.1869
-------------+------------------------------           Adj R-squared =  0.1702
       Total |  5.24541764   199  .026358883           Root MSE      =   .1479

------------------------------------------------------------------------------
       lnSBP |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       woman |   .0035403   .0212026     0.17   0.868    -.0382757    .0453562
             |
    agegrp3 |
          1  |   .0715136   .0253373     2.82   0.005     .0215432     .121484
          2  |    .130465   .0280521     4.65   0.000     .0751404    .1857895
             |
    lnBMI25 |   .2898622   .0772432     3.75   0.000     .1375229    .4422015
      _cons |   4.789641   .0224814   213.05   0.000     4.745303    4.833979
------------------------------------------------------------------------------
```

Note 0 is missing: it is base/reference group

Categorical variable : age group 0 reference

$$\ln(sbp) = \alpha_0 + \alpha_1 \cdot age1 + \alpha_2 \cdot age2 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

Adjusted difference between for a person in **age group 1** compared to **age group 0**

```
------------------------------------------------------------------------------
       lnSBP |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       woman |   .0035403   .0212026     0.17   0.868    -.0382757    .0453562
             |
    agegrp3 |
          1  |   .0715136   .0253373     2.82   0.005     .0215432     .121484
          2  |    .130465   .0280521     4.65   0.000     .0751404    .1857895
             |
    lnBMI25 |   .2898622   .0772432     3.75   0.000     .1375229    .4422015
      _cons |   4.789641   .0224814   213.05   0.000     4.745303    4.833979
------------------------------------------------------------------------------
```

Adjusted difference between for a person in **age group 2** compared to **age group 0**

Expected value for a **man** in **age group 0** with bmi=25.

The expected values:

$age < 40$: $\ln(sbp) = \alpha_0 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln(bmi/25)$

$40 \le age < 50$: $\ln(sbp) = \alpha_0 + \alpha_1 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln(bmi/25)$

$50 \le age$: $\ln(sbp) = \alpha_0 + \alpha_2 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln(bmi/25)$

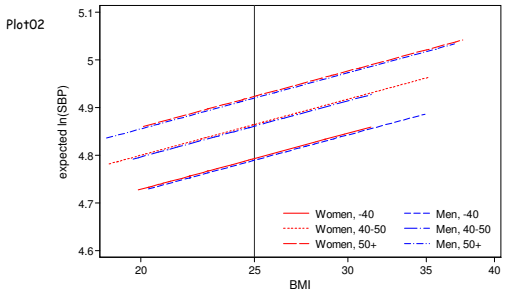
The estimates

lnSBP	Coef.	
woman	.003540	3
1.agegrp3	**.071514**	**1**
2.agegrp3	**.130465**	**2**
lnBMI25	.289862	4
_cons	4.789641	0

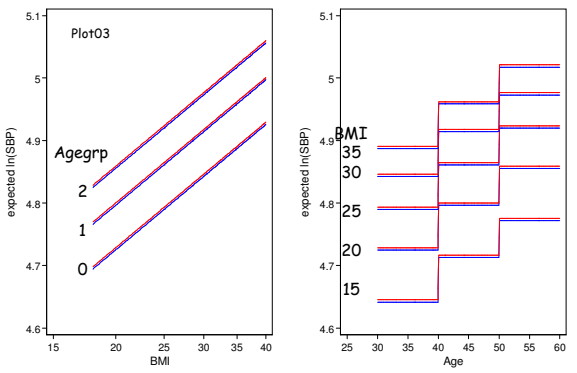
$age < 40$: $\ln(sbp) = 4.789 + 0.004 \cdot woman + 0.290 \cdot \ln(bmi/25)$

$40 \le age < 50$: $\ln(sbp) = 4.789 + 0.072 + 0.004 \cdot woman + 0.290 \cdot \ln(bmi/25)$

$50 \le age$: $\ln(sbp) = 4.789 + 0.130 + 0.004 \cdot woman + 0.290 \cdot \ln(bmi/25)$



Agegroup	Women	Men
0:	$4.793 + 0.290 \cdot \ln(bmi/25)$	$4.790 + 0.290 \cdot \ln(bmi/25)$
1:	$4.865 + 0.290 \cdot \ln(bmi/25)$	$4.861 + 0.290 \cdot \ln(bmi/25)$
2:	$4.924 + 0.290 \cdot \ln(bmi/25)$	$4.920 + 0.290 \cdot \ln(bmi/25)$



Categorical variable : age group 1 reference

$$\ln(sbp) = \gamma_0 + \gamma_1 \cdot age0 + \gamma_2 \cdot age2 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

age0 is one if a person is in age group **0** and zero otherwise

age2 is one if a person is in age group **2** and zero otherwise

The expected $\ln(sbp)$ in the three age groups will be:

$age < 40$: $\ln(sbp) = \gamma_0 + \gamma_1 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln(bmi/25)$

$40 \le age < 50$: $\ln(sbp) = \gamma_0 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln(bmi/25)$

$50 \le age$: $\ln(sbp) = \gamma_0 + \gamma_2 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln(bmi/25)$

We see that γ_1 is the adjusted difference in $\ln(sbp)$ when comparing a person in the **first group** with one in the **second group**.

And γ_2 is the adjusted difference in $\ln(sbp)$ when comparing a person in the **third group** with one in the **second group**.

Categorical variable : age group 1 reference

$$\ln(sbp) = \gamma_0 + \gamma_1 \cdot age0 + \gamma_2 \cdot age2 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

Finally we see that γ_0 is the expected $\ln(sbp)$ for a man in the **second** (reference) age group, with bmi=25.

Many programs (but regression in SPSS) let you chose the reference group

In Stata 11 this done like this:

```
regress lnSBP woman b1.agegrp3 lnBMI25
```

This is categorical with base/reference set to 1

Categorical variable : age group 1 reference

$$\ln(sbp) = \gamma_0 + \gamma_1 \cdot age0 + \gamma_2 \cdot age2 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

Source	SS	df	MS	Number of obs =	200
Model	.980169926	4	.245042482	F(4, 195) =	11.20
Residual	4.26524771	195	.021873065	Prob > F =	0.0000
Total	5.24541764	199	.026358883	R-squared =	0.1869
				Adj R-squared =	0.1702
				Root MSE =	.1479

lnSBP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
woman	.0035403	.0212026	0.17	0.868	-.0382757	.0453562
agegrp3						
0	-.0715136	.0253373	-2.82	0.005	-.121484	-.0215432
2	.0589513	.0263496	2.24	0.026	.0069846	.1109181
lnBMI25	.2898622	.0772432	3.75	0.000	.1375229	.4422015
_cons	4.861154	.0207406	234.38	0.000	4.82025	4.902059

Note 1 is missing: it is base/reference group

Categorical variable : age group 1 reference

$$\ln(sbp) = \gamma_0 + \gamma_1 \cdot age0 + \gamma_2 \cdot age2 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

Adjusted difference between for a person in **age group 0** compared to **age group 1**

lnSBP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
woman	.0035403	.0212026	0.17	0.868	-.0382757	.0453562
agegrp3						
0	-.0715136	.0253373	-2.82	0.005	-.121484	-.0215432
2	.0589513	.0263496	2.24	0.026	.0069846	.1109181
lnBMI25	.2898622	.0772432	3.75	0.000	.1375229	.4422015
_cons	4.861154	.0207406	234.38	0.000	4.82025	4.902059

Adjusted difference between for a person in **age group 2** compared to **age group 1**

Expected value for a man in **age group 1** with *bmi*=25.

Categorical variable: Comparing to parameterizations

$$\ln(sbp) = \alpha_0 + \alpha_1 \cdot age1 + \alpha_2 \cdot age2 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

$$\ln(sbp) = \gamma_0 + \gamma_1 \cdot age0 + \gamma_2 \cdot age2 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

age group 0: $\alpha_0 = \gamma_0 + \gamma_1$

age group 1: $\alpha_0 + \alpha_1 = \gamma_0$

age group 2: $\alpha_0 + \alpha_2 = \gamma_0 + \gamma_2$

$$\alpha_0 = \gamma_0 + \gamma_1 \qquad \gamma_0 = \alpha_0 + \alpha_1$$
$$\alpha_1 = -\gamma_1 \qquad \gamma_1 = -\alpha_1$$
$$\alpha_2 = \gamma_2 - \gamma_1 \qquad \gamma_2 = \alpha_2 - \alpha_1$$
$$\alpha_3 = \gamma_3 \qquad \gamma_3 = \alpha_3$$
$$\alpha_4 = \gamma_4 \qquad \gamma_4 = \alpha_4$$

Categorical variable: Comparing two parameterizations

The estimates:

age group 0 reference

Root MSE = .1479		
lnSBP	Coef.	[95% CI]
woman	.0035	-.0382 .0453
agegrp3		
1	.0715	.0215 .1214
2	.1304	.0751 .1857
lnBMI25	.2898	.1375 .4422
_cons	4.7896	4.745 4.833

age group 1 reference

Root MSE = .1479		
lnSBP	Coef.	[95% CI]
woman	.0035	-.0382 .0453
agegrp3		
0	-.0715	-.1214 -.0215
2	.0589	.0069 .1109
lnBMI25	.2898	.1375 .4422
_cons	4.8611	4.820 4.902

Note, the estimates fulfil the same equations.

The interpretation of the "**agegrp3 2** line" and "**_cons** line" are altered!!!!!!!!!

Always remember: what is the reference group!

Categorical variable : age group 1 reference

lnSBP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
woman	.0035403	.0212026	0.17	0.868	-.0382757	.0453562
agegrp3						
0	-.0715136	.0253373	-2.82	0.005	-.121484	-.0215432
2	.0589513	.0263496	2.24	0.026	.0069846	.1109181
lnBMI25	.2898622	.0772432	3.75	0.000	.1375229	.4422015
_cons	4.861154	.0207406	234.38	0.000	4.82025	4.902059

Two test:

One testing no difference between age group **0** and **1**.
One testing no difference between age group **2** and **1**.

Can we get **one test** testing no difference between age groups?

A F-test in Stata: *testparm* i.agegrp

```
( 1)  0.agegrp3 = 0
( 2)  2.agegrp3 = 0
       F(  2,   195) =   10.93
            Prob > F =    0.0000
```

Highly significant

Interactions/effectmodification

$$\ln(sbp) = \gamma_0 + \gamma_1 \cdot age0 + \gamma_2 \cdot age2 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

One of the **central** assumptions was "no effect modification".

E.g. in model above the "effect" of **age**, **sex** and **bmi** did not depend on the value of each other.

One can **introduce** effect modification between a categorical variable and another variable.

Here we first will look at **agegr3** and **lnBMI25**.

The effect modification will be that the coefficient to **lnBMI25** depend on **age group**.

That is, we will allow **different effect** of bmi in the **different age** groups.

Interactions/effectmodification

$$\ln(sbp) = \omega_0 + \omega_1 \cdot age0 + \omega_2 \cdot age2 + \omega_3 \cdot woman + \omega_4 \cdot \ln\left(\frac{bmi}{25}\right) + \omega_5 \cdot age0 \cdot \ln\left(\frac{bmi}{25}\right) + \omega_6 \cdot age2 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

$age \le 40$: $\ln(sbp) = (\omega_0 + \omega_1) + (\omega_4 + \omega_5) \cdot \ln\left(\frac{bmi}{25}\right) + \omega_3 \cdot woman$

$40 < age \le 50$: $\ln(sbp) = \omega_0 + \omega_4 \cdot \ln\left(\frac{bmi}{25}\right) + \omega_3 \cdot woman$

$50 < age$: $\ln(sbp) = (\omega_0 + \omega_2) + (\omega_4 + \omega_6) \cdot \ln\left(\frac{bmi}{25}\right) + \omega_3 \cdot woman$

ω_1 is the **difference** between the constant for age **group 0** and **reference** group.
ω_5 is the **difference** between the coefficient to **lnBMI25** for age **group 0** and **reference** group.

Interactions/effectmodification

```
. regress lnSBP woman b1.agegrp3##c.lnBMI25
      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  6,   193) =    7.53
       Model |  .994860827     6  .165810138           Prob > F      =  0.0000
    Residual|  4.25055681   193  .02202361            R-squared     =  0.1897
-------------+------------------------------           Adj R-squared =  0.1645
       Total |  5.24541764   199  .026358883           Root MSE      =  .1484

------------------------------------------------------------------------------
      lnSBP |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
------------+-----------------------------------------------------------------
      woman |   .0076438   .0219199     0.35   0.728    -.0355896    .0508772
            |
   agegrp3 |
          0 |  -.0708045   .0261198    -2.71   0.007    -.1223213   -.0192877
          2 |   .0631082   .0270342     2.33   0.021     .0097877    .1164287
            |
   lnBMI25 |   .3155479   .1222905     2.58   0.011     .0743505    .5567453
            |
agegrp3#|
c.lnBMI25|
          0 |   .0429736   .1912373     0.22   0.822    -.3342099     .420157
          2 |  -.1165375   .1855477    -0.63   0.531    -.4824991    .2494242
            |
      _cons |   4.859743   .0214122   226.96   0.000     4.817511    4.901975
------------------------------------------------------------------------------
```


Interactions/effect modification

Ref: constant and 'slope' in reference group

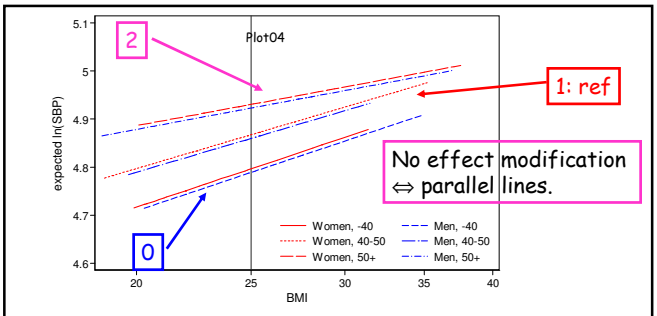
0: difference in constant and slope compared to reference

2: difference in constant and slope compared to reference

lnSBP	Coef.	Std. E.	t	P>|t|	[95% Conf. Interval]	
woman	.0076438	.0219199	0.35	0.728	-.0355896	.0508772	
agegrp3						
0	-.0708045	.0261198	-2.71	0.007	-.1223213	-.0192877	
2	.0631082	.0270342	2.33	0.021	.0097877	.1164287	
lnBMI25	.3155479	.1222905	2.58	0.011	.0743505	.5567453	Ref
agegrp3#c.lnBMI25						
0	.0429736	.1912373	0.22	0.822	-.3342099	.420157	
2	-.1165375	.1855477	-0.63	0.531	-.4824991	.2494242	
_cons	4.859743	.0214122	226.96	0.000	4.817511	4.901975	Ref

Note the larger standard errors

Based on the estimates one can find the six "dose-response" curves:



Agegroup	Women	Men
0:	$4.797 + 0.359 \cdot \ln(bmi/25)$	$4.789 + 0.359 \cdot \ln(bmi/25)$
1:	$4.867 + 0.316 \cdot \ln(bmi/25)$	$4.860 + 0.316 \cdot \ln(bmi/25)$
2:	$4.930 + 0.199 \cdot \ln(bmi/25)$	$4.923 + 0.199 \cdot \ln(bmi/25)$

Interactions/effect modification

lnSBP	Coef.	Std. E.	t	P>|t|	[95% Conf. Interval]
woman	.0076438	.0219199	0.35	0.728	-.0355896 .0508772
agegrp3					
0	-.0708045	.0261198	-2.71	0.007	-.1223213 -.0192877
2	.0631082	.0270342	2.33	0.021	.0097877 .1164287
lnBMI25	.3155479	.1222905	2.58	0.011	.0743505 .5567453
agegrp3#c.lnBMI25					
0	.0429736	.1912373	0.22	0.822	-.3342099 .420157
2	-.1165375	.1855477	-0.63	0.531	-.4824991 .2494242
_cons	4.859743	.0214122	226.96	0.000	4.817511 4.901975

Two tests:

One testing differences between "slope" in age group **0** and **1**.

One testing differences between "slope" in age group **2** and **1**.

One **test** testing no difference between age groups!

A F-test in Stata: *testparm* `i.agegrp3#c.lnBMI25`

```
 ( 1)  0.agegrp3#c.lnBMI25 = 0
 ( 2)  2.agegrp3#c.lnBMI25 = 0
       F(  2,   193) =    0.33
            Prob > F =    0.7168
```

Non-significant

Interactions/effect modification

The test of no interaction was non-significant.

But look at the **confidence interval** for the difference in slope for between age group **2** and group **1** !

lnSBP	Coef.	Std. E.	t	P>|t|	[95% Conf. Interval]
woman	.0076438	.0219199	0.35	0.728	-.0355896 .0508772
agegrp3					
0	-.0708045	.0261198	-2.71	0.007	-.1223213 -.0192877
2	.0631082	.0270342	2.33	0.021	.0097877 .1164287
lnBMI25	.3155479	.1222905	2.58	0.011	.0743505 .5567453
agegrp3#c.lnBMI25					
0	.0429736	.1912373	0.22	0.822	-.3342099 .420157
2	-.1165375	.1855477	-0.63	0.531	-.4824991 .2494242
_cons	4.859743	.0214122	226.96	0.000	4.817511 4.901975

It is very wide!!! We know very little about this difference!

The test for no interaction has **very low power**!!!

The data have very **little information** on whether there is effect modification.

Interaction between age group and sex

regress lnSBP lnBMI25 **b1.agegrp3##b1.sex** Male sex==1

lnSBP	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]
lnBMI25	.2265018	.0774898	2.92	0.004	.0736662 .3793374
agegrp3					
0	-.0426734	.0377096	-1.13	0.259	-.1170493 .0317025
2	-.0025412	.0365457	-0.07	0.945	-.0746215 .0695391
2.sex	-.0210869	.0322283	-0.65	0.514	-.0846518 .042478
agegrp3#sex					
0 2	-.0548967	.0501668	-1.09	0.275	-.1538422 .0440488
2 2	.133379	.0501308	2.66	0.008	.0345043 .2322536
_cons	4.873442	.0251767	193.57	0.000	4.823786 4.923099

```
. testparm i.agegrp3#i.sex

 ( 1)  0.agegrp3#2.sex = 0
 ( 2)  2.agegrp3#2.sex = 0

       F(  2,   193) =    6.26
            Prob > F =    0.0023
```

Highly significant

Interaction between age group and sex

Differences between age groups among men are small

lnSBP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnBMI25	.2265018	.0774898	2.92	0.004	.0736662	.3793374
agegrp3						
0	-.0426734	.0377096	-1.13	0.259	-.1170493	.0317025
2	-.0025412	.0365457	-0.07	0.945	-.0746215	.0695391
2.sex	-.0210869	.0322283	-0.65	0.514	-.0846518	.042478
agegrp3#sex						
0 2	-.0548967	.0501668	-1.09	0.275	-.1538422	.0440488
2 2	.133379	.0501308	2.66	0.008	.0345043	.2322536
_cons	4.873442	.0251767	193.57	0.000	4.823786	4.923099

Women age group 1: 4.873–0.021=4.852

Women age group 0: 4.873–0.021–0.042–0.055=4.755 (−0.097)

Women age group 2: 4.873–0.021–0.003+0.133=4.982 (0.130)

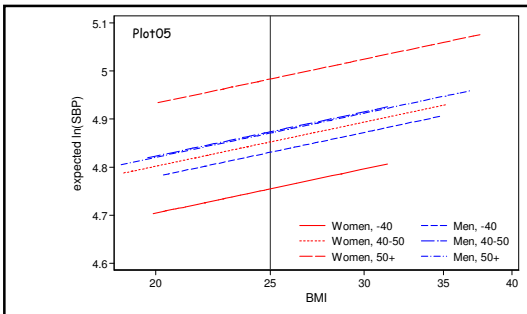
Large differences in the age groups among women.

Interaction between age group and sex

Using women as reference: b2.sex

Large differences between age groups among women

lnSBP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnBMI25	.2265018	.0774898	2.92	0.004	.0736662	.3793374
agegrp3						
0	-.0975701	.0328469	-2.97	0.003	-.162355	-.0327852
2	.1308378	.0354804	3.69	0.000	.0608587	.2008168
1.sex	.0210869	.0322283	0.65	0.514	-.042478	.0846518
agegrp3#sex						
0 1	.0548967	.0501668	1.09	0.275	-.0440488	.1538422
2 1	-.133379	.0501308	-2.66	0.008	-.2322536	-.0345043
_cons	4.852355	.0205502	236.12	0.000	4.811824	4.892887



Agegroup	Women	Men
0:	$4.755+0.227\cdot\ln(bmi/25)$	$4.831+0.227\cdot\ln(bmi/25)$
1:	$4.852+0.227\cdot\ln(bmi/25)$	$4.873+0.227\cdot\ln(bmi/25)$
2:	$4.982+0.227\cdot\ln(bmi/25)$	$4.871+0.227\cdot\ln(bmi/25)$