

Logistic regression

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Stata 11

When one might use logistic regression.

Some examples:

- One **binary** independent variable. (**one odds ratio**).
- Probabilities, odds and the logit function
- One **continuous** independent variable.
- One **categorical** independent variable.
(The **Wald test**)
- One **binary** independent variable and **continuous** independent variable no interaction.
- One **binary** independent variable and **continuous** independent variable with interaction.

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Linear and Logistic regression - Note 4

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Watch out for 'small' **reference** groupsThe **likelihood ratio test**: comparing two nested models.The **logistic regression model in general**The model and the **assumptions**.The **data** and the assumption of **independence**.Estimation and **inference**

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Linear and Logistic regression - Note 4

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Logistic regression models: Introduction

A logistic regression is a **possible** model if the **dependent** variable (the response) is **dichotomous** dead/alive obese/not obese etc.

Contrary to what many believe there are **no assumptions** about the **independent** variables.

They can be categorical or continuous.

When working with binary response it is **custom** to code the "positive" event (eg. dead) as **1** and a "negative" event (alive) as **0**.

A logistic regression models the **probability** of a "positive event" via odds.

And the associations via **odds ratio**.

If the **event is rare** then **odds ratios** estimate the **relative risk**.

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Linear and Logistic regression - Note 4

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Logistic regression models: Introduction

A logistic regression can also be used to estimate the odds ratios in an **unmatched case-control** study.

For such data the **constant terms** have **no meaning**.

And the odds ratios is comparable to the odds ratio from a **follow-up study**.

Many **other epidemiological design** are analyzed by logistic regression models.

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Linear and Logistic regression - Note 4

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Estimating one odds ratio using logistic regression

We are now considering a larger part of the Frammingham data set, consisting of 4690 persons with **known BMI** at the start.

We will focus on the risk obesity (BMI \geq 30 kg/m 2).

Out of the 4690 persons 601 = 12.8% were **obese**.

Divided into gender

	Obese	Not-Obese
Women	375 (14.2%)	2268
Men	226 (11.0%)	1821

We see a higher prevalence among women: OR: 1.33 (1.12;1.59).

That is the **odds** of being obese is between 12 and 59 percent higher for women. ($\chi^2=10.2$ p-value=0.001)

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Finding an odds ratio using logistic regression

The odds ratio is defined as: $OR = \frac{odds_{Women}}{odds_{Men}}$

So applying the logarithm we get:

$$\ln(OR) = \ln\left(\frac{odds_{Women}}{odds_{Men}}\right) = \ln(odds_{Women}) - \ln(odds_{Men})$$

And rearranging terms :

$$\ln(odds_{Women}) = \ln(odds_{Men}) + \ln(OR)$$

That is the log-odds obesity for the women can be written as the sum of two terms:

- The log-odds in **reference** group (men)
- The log of the odds ratio

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Finding an odds ratio using logistic regression

$$\ln(\text{odds}_{\text{Women}}) = \ln(\text{odds}_{\text{Men}}) + \ln(\text{OR})$$

If we again let *women* be an indicator/dummy variable, then we can consider the model:

$$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman}$$

For *men* we get: $\ln(\text{odds}) = \beta_0$

And for *women*: $\ln(\text{odds}) = \beta_0 + \beta_1$

Comparing with the equation on top we get:

$$\beta_0 = \ln(\text{odds}_{\text{Men}})$$

and

$$\beta_1 = \ln(\text{OR})$$

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Finding an odds ratio using logistic regression

$$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman}$$

$$\ln(\text{odds}_{\text{Men}}) \quad \ln(\text{OR})$$

Or to be more precise: $\beta_1 = \ln(\text{OR}_{\text{Women vs Men}})$

So, if we can fit the model above to the data, then we can get an estimate of the $\ln(\text{OR})$ and hence of OR !

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Probabilities and odds

If p denotes the probability of an event (the **risk**, the **prevalence** proportion, or **cumulated incidence** proportion) then the odds is given by:

$$\text{odds} = \frac{p}{1-p}$$

Note: $\text{odds}=1 \Leftrightarrow p=0.5 \Leftrightarrow \ln(\text{odds})=0$

$$\ln(\text{odds}) = \ln\left(\frac{p}{1-p}\right)$$

In mathematics the last function of p is called the "logit" function.

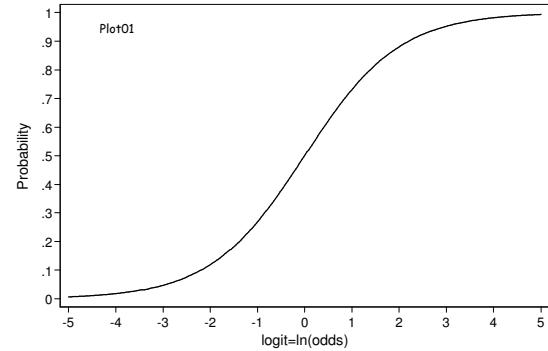
$$\text{logit}(p) = \ln\left(\frac{p}{1-p}\right)$$

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Probabilities and odds



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Probabilities and odds

$$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman}$$

So modelling the **log-odds** is the same as modelling $\text{logit}(p)$ and model from before could be written.

$$\text{logit}(p) = \beta_0 + \beta_1 \cdot \text{woman}$$

Going from odds to probabilities: $p = \frac{\text{odds}}{1+\text{odds}}$

The model on **probability scale** is :

$$p = \frac{\exp(\beta_0 + \beta_1 \cdot \text{woman})}{1 + \exp(\beta_0 + \beta_1 \cdot \text{woman})} = \text{INVLOGIT}(\beta_0 + \beta_1 \cdot \text{woman})$$

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Finding an odds ratio using logistic regression

$$\text{logit}(p) = \ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman}$$

Back to finding the estimates.

In Stata: `logit obese b1.sex, baselevel`

Iteration 0: log likelihood = -1795.5437

Iteration 1: log likelihood = -1790.3856

Iteration 2: log likelihood = -1790.3703

Iteration 3: log likelihood = -1790.3703

Logistic regression

Number of obs = 4690

LR chi2(1) = 10.35

Prob > chi2 = 0.0013

Pseudo R2 = 0.0029

obese	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
sex	(base)				
1	.2868784	.0898972	3.19	0.001	.1106831 .4630738
2					
_cons	-2.086606	.0705261	-29.59	0.000	-2.224835 -1.948378

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Linear and Logistic regression - Note 4

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Finding an odds ratio using logistic regression

$$\text{logit}(p) = \ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman}$$

$$\hat{\beta}_1 = \ln(\widehat{OR})$$

95% CI for $\ln(OR)$

obese	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
2	0.2868784	0.0898972	3.19	0.001	1.1106831 0.4630738
_cons	-2.086606	0.070526	-29.59	0.000	-2.224835 -1.948378

$$OR = \exp(0.2868784) = 1.33$$

95% CI: (1.12; 1.59).

Test for the hypothesis: $\ln(OR)=0 \Leftrightarrow OR=1$

Odds in reference group (men) = $\exp(-2.086606) = 0.1241$

95% CI: (0.1081; 0.1425).

Prevalence among men: 0.1104 (0.0975; 0.1247).

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Finding an odds ratio using logistic regression

$$\text{logit}(p) = \ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman}$$

An easier way to obtain the odds ratio.
`logit obese b1.sex, or baselevel`

Iteration 0: log likelihood = -1795.5437
 Iteration 3: log likelihood = -1790.3703
 Logit estimates
 Number of obs = 4690
 LR chi2(1) = 10.35
 Prob > chi2 = 0.0013
 Pseudo R2 = 0.0029
 Log likelihood = -1790.3703

obese	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
sex	(base)				
1	1.332262	0.1197667	3.19	0.001	1.117041 1.588951
2					

Note, we cannot find any information about the risk in the reference group, i.e. the odds and prevalence among men!

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Linear and Logistic regression - Note 4

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The obesity and age: version 1

In the previous section we saw that the prevalence of obesity was different between men and women.

Is it also associated with age?

The simplest model on the **logit scale** would be:

$$\text{logit}(p) = \ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{age}$$

That is a linear relation on the log-odds scale.

As we have seen before using **age** implies that β_0 references to a newborn ($\text{age}=0$).

So we will choose $\text{age}=45$ reference instead:

$$\text{logit}(p) = \ln(\text{odds}) = \beta_0 + \beta_1 \cdot (\text{age} - 45)$$

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The obesity and age: version 1

$$\text{logit}(p) = \ln(\text{odds}) = \beta_0 + \beta_1 \cdot (\text{age} - 45)$$

The interpretation of the parameters:

β_0 : the **log odds** for a 45 year old person.

β_1 : the **log odds ratio**, when comparing two persons who differ 1 year in age.

$\exp(\beta_1)$: the **odds ratio**, when comparing two persons who differ 1 year in age.

Note, that this odds ratio is **assumed** to be the same no matter what age the two persons have, as long as they differ by one year!

The log odds ratio is **proportional** to the age differences, e.g. OR increases **exponentially** with the age differences.

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The obesity and age: version 1

$$\text{logit}(p) = \ln(\text{odds}) = \beta_0 + \beta_1 \cdot (\text{age} - 45)$$

Obtaining the estimates in Stata:

$$\text{generate age45}=\text{age}-45$$

$$\text{logit obese age45}$$

obese	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
age45	-0.0348023	.0051296	6.78	0.000	.0247484 .0448561
_cons	-1.985922	.0463594	-42.84	0.000	-2.076785 -1.895059

Test for no association with **age**

$$\text{logit obese age45, OR}$$

obese	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
age45	1.035415	0.053113	6.78	0.000	1.025057 1.045877

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The obesity and age: version 1

$$\text{logit}(p) = \ln(\text{odds}) = \beta_0 + \beta_1 \cdot (\text{age} - 45)$$

Estimate: $\beta_0 = -1.985$ (-2.0767; -1.8951)

The **odds** for obesity among 45 year old:

$$0.1373 (0.1253; 0.1503)$$

The **prevalence** of obesity among 45 year old:

$$0.1207 (0.1114; 0.1307)$$

$odds = \exp(\log(odds))$

$Prob = \frac{odds}{1+odds}$

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The obesity and age: version 1

$\text{logit}(p) = \ln(\text{odds}) = \beta_0 + \beta_1 \cdot (\text{age} - 45)$

Estimates: $\beta_1 : 0.0348 (0.0247; 0.0449)$

The odds ratio for being obese is 1.0354 (1.0251; 1.0459) when comparing the old person to the young person, if they differ with one year in age.

If they differ with 4.5 years then the odds ratio is 1.0354^{4.5} (1.0251^{4.5}; 1.0459^{4.5}) = 1.17 (1.12; 1.22)

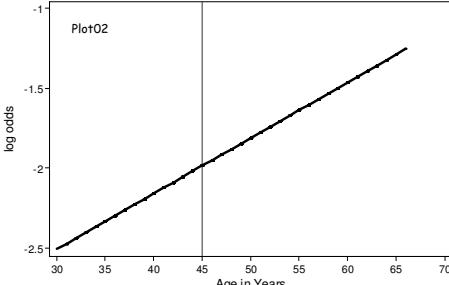
In Stata: `lincom age45*4.5, OR`

```
( 1) 4.5 age45 = 0
obese | Odds Ratio Std. Err. z P>|z| [95% Conf. Interval]
(1) | 1.16954 .0269568 6.78 0.000 1.117806 1.223668
```

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The obesity and age: version 1

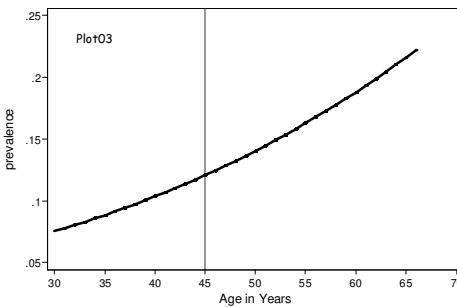
Estimated relationship: $\ln(\text{odds}) = -1.986 + 0.0348 \cdot (\text{age} - 45)$



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The obesity and age: version 1

Estimated relationship:

$$\text{prevalence} = \frac{\exp(-1.986 + 0.0348 \cdot (\text{age} - 45))}{1 + \exp(-1.986 + 0.0348 \cdot (\text{age} - 45))}$$


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The obesity and age: version 2

$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot (\text{age} - 45)$

This model assumes that one year of age difference is associated with the same odds ratio irrespectively of the age.

An other way to model the prevalence could be to assume a step function that is to categorize age.

We will here look at age divided in seven five-years groups: `egen agegrp7=cut(age), at(0,35,40,45,50,55,60,120) tab7`

With this command the youngest age group will be number 0 the second youngest: 1 and the oldest: 6

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The obesity and age: version 2

table agegrp7 ,c(min age max age count obese sum obese) row

agegrp7	min(age)	max(age)	N(obese)	sum(obese)
0-	30	34	352	23
35-	35	39	973	105
40-	40	44	885	93
45-	45	49	799	95
50-	50	54	733	115
55-	55	59	613	95
60-	60	66	335	75
Total	30	66	4,690	601

A model that have different odds in each age group :

$$\ln(\text{odds}) = \alpha_0 + \sum_{i=1}^6 \alpha_i \cdot \text{age}_i$$

Where age_i is an indicator for being in the i th age group

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The obesity and age: version 2

$$\ln(\text{odds}) = \alpha_0 + \sum_{i=1}^6 \alpha_i \cdot \text{age}_i$$

The interpretation of the parameters:

α_0 : the log odds in reference group=the youngest.

α_i : the log odds ratio, when comparing one person in age group i with one in the reference group=the youngest.

Not all output

logit obese i.agegrp7,baselevel

obese	coef.	Std. Err.	z	P> z	[95% Conf. Interval]
agegrp7					
0 (base)					
1	.5483322	.239152	2.29	0.022	.0796029 1.017061
2	.5186016	.2419361	2.14	0.032	.0444155 .9927877
3	.6576621	.2417944	2.72	0.007	.1837537 1.13157
4	.9790072	.2383937	4.11	0.000	.5117642 1.44625
5	.9644652	.2428468	3.97	0.000	.4884941 1.440436
6	1.41737	.2523832	5.62	0.000	.9227081 1.912032
_cons	-2.660564	.2156798	-12.34	0.000	-3.083288 -2.237839

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The obesity and age: version 2

$$\ln(\text{odds}) = \alpha_0 + \sum_{i=1}^6 \beta_i \cdot \text{age}_i$$

Not all output

```
logit obese i.agegrp7, or baselevel
```

	obese	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
1	1.730365	.1138201	2.29	0.022	1.082857	2.765057
2	1.679677	.4063746	2.14	0.032	1.045417	2.698747
3	1.930274	.4661295	2.72	0.007	1.20172	3.100522
4	2.661812	.6145592	4.11	0.000	1.668232	4.247159
5	2.623384	.6370106	3.97	0.000	1.62986	4.222538
6	4.126254	.041397	5.62	0.000	2.516095	6.766825

The OR between the second oldest and the youngest: 2.62 (1.63;4.22)

Between a 63 and 322 percent increase in odds.

Small prevalence: 63 and 322 percent increase in prevalence.

A statistical significant difference in prevalence!

The obesity and age: version 2

$$\ln(\text{odds}) = \alpha_0 + \sum_{i=1}^6 \alpha_i \cdot \text{age}_i$$

The output contains six tests of no difference in risk - comparing each of the six groups with the reference (the youngest) group.

The command: `testparm i.agegrp7` will give a "Wald test" of no difference between the seven groups.

```
( 1 ) [obese]1.agegrp7 = 0
( 2 ) [obese]2.agegrp7 = 0
( 3 ) [obese]3.agegrp7 = 0
( 4 ) [obese]4.agegrp7 = 0
( 5 ) [obese]5.agegrp7 = 0
( 6 ) [obese]6.agegrp7 = 0
```

chi2(6) = 55.26
Prob > chi2 = 0.0000

Highly significant differences

The obesity and age: version 2

Using the age group 45-49 as reference

```
logit obese b3.agegrp7, or baselevel
```

Not all output

	obese	Odds Ratio	Std. Err.	z	P> z	[95% Conf. Interval]
agegrp7						
0	.5180611	.1252643	-2.72	0.007	.3225264	.8321407
1	.8964346	.1347312	-0.73	0.467	.6675609	1.203778
2	.8701754	.1347005	-0.90	0.369	.6424561	1.17861
3	(base)					
4	1.378981	.2057436	2.15	0.031	1.029341	1.847385
5	1.359073	.2123097	1.96	0.050	1.000625	1.845927
6	2.137652	.3648206	4.45	0.000	1.529915	2.986803

The OR between the second oldest and the 45-49 old: 1.36 (1.00;1.85)

Between a no and 85 percent increase in (odds) prevalence.

A borderline significant different in prevalence!

The obesity and age: version 2

Estimated relationship

The obesity and age: version 1 and 2

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The obesity, sex and age: version 1

The first analysis only looked at sex and the second only at age.

Let us try to look at those two at the same time

The simplest model on the logit scale would be:

$$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman} + \beta_2 \cdot (\text{age} - 45)$$

This is based on three assumptions:

Additivity on logit scale: The contribution from sex and age are added.

Proportionality on logit scale: The contribution from age is proportional to its value.

No effectmodification on logit scale: The contribution from one independent variable is the same whatever the value is for the other.

The obesity, sex and age : version 1

$$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman} + \beta_2 \cdot (\text{age} - 45)$$

The interpretation of the parameters:

β_0 : the log odds for a 45 year old man.

β_1 : the log odds ratio, when comparing a woman to a man of the same age.

β_2 : the log odds ratio, when comparing two persons of the same sex, where the first is one year older than the other.

$\beta_2 \cdot \Delta \text{age}$: the log odds ratio, when comparing two persons of the same sex, where the first is Δage years older than the other.

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The obesity, sex and age : version 1

$$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman} + \beta_2 \cdot (\text{age} - 45)$$

Obtaining the estimates in Stata:

logit obese b1.sex age45

Iteration 0: log likelihood = -1795.5437

Iteration 3: log likelihood = -1767.7019

Logistic regression

		Number of obs	=	4690		
		LR chi2(2)	=	55.68		
		Prob > chi2	=	0.0000		
		Pseudo R2	=	0.0155		
obese		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
sex						
1		(base)				
2		.2439376	.0903385	3.04	0.002	.0973374 .4514579
age45		.0344723	.0051354	6.71	0.000	.0244072 .0445374
_cons		-2.147056	.0721961	-29.74	0.000	-2.388561 -2.00555

Tests: No association with sex No association with age

Prevalence is 50% among 45 year old men

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The obesity, sex and age : version 1

$$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman} + \beta_2 \cdot (\text{age} - 45)$$

logit obese b1.sex age45, or
obese | Odds Ratio Std. Err. z P>|z| [95% Conf. Interval]

2.sex		1.315738	.1188618	3.04	0.002
age45		1.035073	.0053155	6.71	0.000

OR for women compared to men "adjusted for age":

1.32 (1.10;1.57)

The unadjusted was 1.33 (1.12;1.59).

OR for one year age difference "adjusted for sex":

1.04 (1.02;1.05)

The unadjusted was 1.04 (1.03;1.05)

Not much has changed!

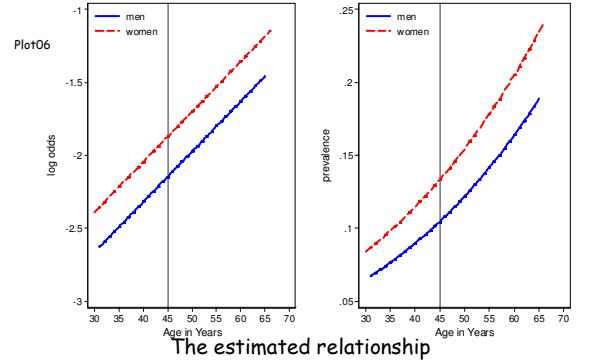
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The obesity, sex and age : version 1

$$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman} + \beta_2 \cdot (\text{age} - 45)$$



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The obesity, sex and age: version 2

A more complicated model on the logit scale would be:

$$\text{men: } \ln(\text{odds}) = \alpha_0 + \alpha_1 \cdot (\text{age} - 45)$$

$$\text{women: } \ln(\text{odds}) = \gamma_0 + \gamma_1 \cdot (\text{age} - 45)$$

This is based on one assumption:

Proportionality on the logit scale: The contribution age is proportional to its value.

It can be written in just one formula (with interaction):

$$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman} + \beta_2 \cdot (\text{age} - 45) + \beta_3 \cdot \text{woman} \cdot (\text{age} - 45)$$

$$\alpha_0 = \beta_0 \quad \alpha_1 = \beta_2$$

$$\text{Where: } \gamma_0 = \beta_0 + \beta_1 \quad \gamma_1 = \beta_2 + \beta_3$$

$$\text{That is: } \beta_1 = \gamma_0 - \alpha_0 \quad \beta_3 = \gamma_1 - \alpha_1$$

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Linear and Logistic regression - Note 4

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The obesity, sex and age: version 2

$$\ln(\text{odds}) = \beta_0 + \beta_1 \cdot \text{woman} + \beta_2 \cdot (\text{age} - 45) + \beta_3 \cdot \text{woman} \cdot (\text{age} - 45)$$

Estimates log odds:

		obese	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
2.sex		.116797	.0950345	1.23	0.219	-.0694672 .3030611	
age45		-.005684	.0083728	-0.68	0.497	-.0220953 .0107255	
sex#c.age45		.2	.065803	6.13	0.000	.0447472 -.0868588	
_cons		-2.083041	.0706433	-29.49	0.000	-2.221499 -.1.944583	

Men

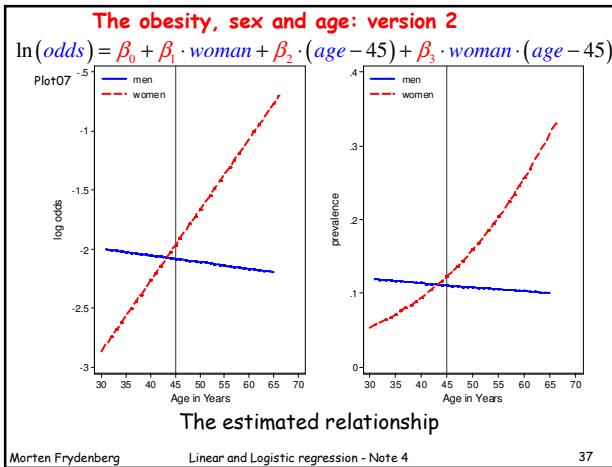
Difference between women and men

Estimates odds ratios:

		obese	Odds Ratio	z	P> z	[95% Conf. Interval]
2.sex		1.123891		1.23	0.219	.9328907 .1.353997
age45		.9943312		-0.68	0.497	.978147 .1.010783
sex#c.age45		.2	1.068016	6.13	0.000	1.045763 .1.090743

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A small case-control example

tabodds cancer age				
age	cases	controls	odds	[95% Conf. Interval]
25-34	2	116	0.01724	0.00426 0.06976
35-44	9	190	0.04737	0.02427 0.09244
45-54	46	167	0.27545	0.19875 0.38175
55-64	76	166	0.45783	0.34899 0.60061
65-74	55	106	0.51887	0.37463 0.71864
>=75	13	31	0.41935	0.21944 0.80138

Few events in reference group= wide CI's

tabodds cancer age, or				
age	Odds Ratio	chi2	P>chi2	[95% Conf. Interval]
25-34	1.00000	1.76	0.1843	0.579474 13.025660
35-44	2.747368	24.18	0.0000	3.588609 71.123412
45-54	15.976048	41.14	0.0000	5.834718 120.850133
55-64	26.554217	43.99	0.0000	6.278745 144.243682
65-74	30.094340	29.40	0.0000	4.402342 134.380270
>=75	24.322581			

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A small case-control example

tabodds cancer age				
age	cases	controls	odds	[95% Conf. Interval]
25-34	2	116	0.01724	0.00426 0.06976
35-44	9	190	0.04737	0.02427 0.09244
45-54	46	167	0.27545	0.19875 0.38175
55-64	76	166	0.45783	0.34899 0.60061
65-74	55	106	0.51887	0.37463 0.71864
>=75	13	31	0.41935	0.21944 0.80138

'Many' events in reference group= narrow CI's

tabodds cancer age, or base(3)				
age	Odds Ratio	chi2	P>chi2	[95% Conf. Interval]
25-34	0.062594	24.18	0.0000	0.014060 0.278660
35-44	0.171968	25.86	0.0000	0.079661 0.371235
45-54	1.000000	.	.	.
55-64	1.662127	5.54	0.0186	1.083844 2.548952
65-74	1.883716	7.32	0.0068	1.181689 3.002809
>=75	1.522440	1.30	0.2546	0.734799 3.154365

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A small case-control example

```
logit cancer b0.smoker b1.age, or
Iteration 0: log likelihood = -496.55682
Iteration 1: log likelihood = -437.36405
Iteration 2: log likelihood = -429.36499
Iteration 3: log likelihood = -428.94718
Iteration 4: log likelihood = -428.94432
Iteration 5: log likelihood = -428.94432
Logistic regression
Number of obs = 977
LR chi2(6) = 135.23
Prob > chi2 = 0.0000
Pseudo R2 = 0.1362
```

cancer Odds Ratio Std. Err. z P> z [95% Conf. Interval]					
smoker	0 (base)	1	.4513038	4.45	0.000 1.613342 3.424472
age	1 (base)	2	2.832192	2.243677	1.31 0.189 .5995101 13.37978
	3	16.58078	12.17376	3.82 0.000 3.932284 69.91412	
	4	27.89911	20.32372	4.57 0.000 6.691354 116.3233	
	5	34.79453	25.59025	4.83 0.000 8.231513 147.0761	
	6	27.713	21.89264	4.21 0.000 5.891876 130.3507	

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A small case-control example

```
logit cancer b0.smoker b3.age, or baselev
Iteration 0: log likelihood = -496.55682
Iteration 1: log likelihood = -437.36405
Iteration 2: log likelihood = -429.36499
Iteration 3: log likelihood = -428.94718
Iteration 4: log likelihood = -428.94432
Iteration 5: log likelihood = -428.94432
Logistic regression
Number of obs = 977
LR chi2(6) = 135.23
Prob > chi2 = 0.0000
Pseudo R2 = 0.1362
```

cancer Odds Ratio Std. Err. z P> z [95% Conf. Interval]					
smoker	0 (base)	1	.4513038	4.45	0.000 1.613342 3.424472
age	1	.0603108	.0442807	-3.82 0.000 .014303 .254305	
	2	.1708118	.0652397	-4.63 0.000 .080800 .361098	
	3 (base)	1.682618	.3701188	2.37 0.018 1.093327 2.58953	
	4	2.098486	.5042862	3.08 0.002 1.31025 3.360918	
	5	1.671393	.6277714	1.37 0.171 .800514 3.489699	

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Things to look out for in the output

In general:

Wide CI's or large standard errors in a logistic regression indicates that at least one group has **few events!**

Many iterations in a logistic regression indicates that some of the **parameters are hard to estimate**.

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Comparing two models: the likelihood ratio test

Earlier we saw how one could use a **Wald** test to test if several coefficients could be zero.

An other way to "compare" two models is by a **likelihood ratio test**.

In the logistic regression output from Stata we find a likelihood ratio test comparing the **fitted model** with the model with no dependent variables the **constant odds model**:

LR chi2(6) = 135.23
Prob > chi2 = 0.0000

The conclusion: The model with **smoker** and **age** is **statistical significant** better, than a model assuming the same odds, risk for everybody.

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Linear and Logistic regression - Note 4

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Comparing two models: the likelihood ratio test

One can compare two models with a likelihood ratio test if:

- The two models are fitted on exactly the **same data set**.
- The two models are **nested**, i.e. one can go from one model to the other by setting some coefficients to zero.

In Stata the test is found in this way:

```
logit cancer i.smoker i.age
estimates store model1
logit cancer i.smoker
estimates store model2
lrtest model1 model2
```

Output:

```
likelihood-ratio test          LR chi2(5) = 120.82
(Assumption: model2 nested in model1)  Prob > chi2 = 0.0000
```

i.age adds statistical significant information to the model only containing smoking!

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Linear and Logistic regression - Note 4

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Logistic regression model in general

$$\ln(\text{odds}) = \beta_0 + \sum_{p=1}^k \beta_p \cdot x_p$$

This is based on three assumptions:

- Additivity on log-odds scale:** The contribution from each of the independent variables are **added**.
- Proportionality:** The contribution from independent variables is **proportional** to its value (with a factor β)
- No effectmodification:** The contribution from one independent variable is **the same** whatever the values are for the other.

Note a. can also be formulate as **multiplicativity on odds scale**

$$\text{odds} = \text{odds}_0 \cdot OR_1^{x_1} \cdot OR_2^{x_2} \cdots OR_k^{x_k}$$

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Linear and Logistic regression - Note 4

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Logistic regression model in general

$$\ln(\text{odds}) = \beta_0 + \sum_{p=1}^k \beta_p \cdot x_p$$

If one consider two persons who differ with

Δx_1 in x_1 , Δx_2 in x_2 ... and Δx_k in x_k

the difference in the **log odds** is :

$$\sum_{p=1}^k \beta_p \cdot \Delta x_p$$

Again we see that the contribution from each of the explanatory variables:

- are **added**,
- are **proportional to the difference**
- and **does not depend** on the difference in the other

on the **log odds scale**.

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Linear and Logistic regression - Note 4

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Logistic regression model in general

$$\ln(\text{odds}) = \beta_0 + \sum_{p=1}^k \beta_p \cdot x_p$$

If one consider two persons who differ with

Δx_1 in x_1 , Δx_2 in x_2 ... and Δx_k in x_k

the odds ratio : $OR = OR_1^{\Delta x_1} \cdot OR_2^{\Delta x_2} \cdots OR_k^{\Delta x_k}$

Note the model might also be formulated:

$$p = \Pr[Y=1] = \frac{\exp\left(\beta_0 + \sum_{p=1}^k \beta_p \cdot x_p\right)}{1 + \exp\left(\beta_0 + \sum_{p=1}^k \beta_p \cdot x_p\right)}$$

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Linear and Logistic regression - Note 4

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Logistic regression model in general

$$\ln(\text{odds}) = \beta_0 + \sum_{p=1}^k \beta_p \cdot x_p$$

The data: $Y=1/0$ dichotomous dependent variable

$x_1, x_2 \dots x_k$ independent/explanatory variables

Like in the normal regression models it is assumed that the Y 's are **independent** given the explanatory variables.

This assumption can, in general, only be checked by **scrutinising** the design.

Look out for data sampled in **clusters**:

Patients within the **same GP**

Children within the **same family**

Twins.

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Linear and Logistic regression - Note 4

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Logistic regression model in general**Estimation:**

Excepting the two by two tables, there are **no closed form** for the estimates.

The **distribution** of the estimates **are not known**.

Estimates are found by the method of **maximum likelihood**.

Estimates are using **iterative methods**.

Standard errors, confidence intervals and all tests are based on **asymptotics**.

That is, all statistical **inference** are **approximate**.

The **more data** - the more events -the **better** the approximations.