Multiple linear regression 2
Stata 11

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Categorical variables in regression models.

The changing **reference** level

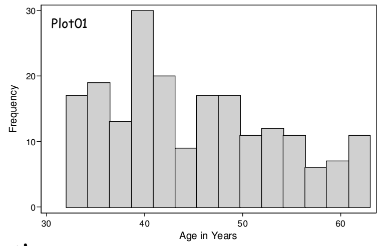
Interaction/effect modification

Interaction between a **categorical** and **continuous** variable

Interaction between two **categorical** variables

Categorical variable in regression models

The age distribution:



Let us divide *age* into **three** agegroups ,

0: *age*≤40, 1: 40<*age*≤50, 2: 50<*age*

and consider the new model

$$\ln(sbp)=\alpha_0+\alpha_1\cdot age1+\alpha_2\cdot age2+\alpha_3\cdot woman+\alpha_4\cdot\ln\left(\frac{bmi}{25}\right)+E$$

Categorical variable : age group 0 reference

$$\ln(sbp)=\alpha_0+\alpha_1\cdot age1+\alpha_2\cdot age2+\alpha_3\cdot woman+\alpha_4\cdot\ln\left(\frac{bmi}{25}\right)+E$$

age1 is one if a person is in age group **1** and zero otherwise
age2 is one if a person is in age group **2** and zero otherwise

The expected $\ln(sbp)$ in the three age groups will be:

$age<40:\quad \ln(sbp)=\alpha_0 \quad +\alpha_3\cdot woman+\alpha_4\cdot\ln(bmi/25)$

$40\le age<50:\quad \ln(sbp)=\alpha_0+\alpha_1 \quad +\alpha_3\cdot woman+\alpha_4\cdot\ln(bmi/25)$

$50\le age:\quad \ln(sbp)=\alpha_0+\alpha_2 \quad +\alpha_3\cdot woman+\alpha_4\cdot\ln(bmi/25)$

We see that α_1 is the adjusted difference in $\ln(sbp)$ when comparing a person in the **second group** with one in the **first group**.

And α_2 is the adjusted difference in $\ln(sbp)$ when comparing a person in the **third group** with one in the **first group**.

Categorical variable : age group 0 reference

$$\ln(sbp)=\alpha_0+\alpha_1\cdot age1+\alpha_2\cdot age2+\alpha_3\cdot woman+\alpha_4\cdot\ln\left(\frac{bmi}{25}\right)+E$$

Finally we see that α_0 is the expected $\ln(sbp)$ for a **man** in the **first** (reference) age group, with bmi=25.

In most programs the model is fitted by first generating the grouping variable and then making the regression telling the program which variables are categorical.

In Stata this done is like this:

```
egen agegrp3=cut(age), at(0,40,50,120) label
  regress lnSBP woman i.agegrp3 lnBMI25
```

This is categorical

Categorical variable : age group 0 reference

$$\ln(sbp)=\alpha_0+\alpha_1\cdot age1+\alpha_2\cdot age2+\alpha_3\cdot woman+\alpha_4\cdot\ln\left(\frac{bmi}{25}\right)+E$$

OUTPUT:

```
  Source |       SS       df       MS              Number of obs =     200
---------+------------------------------           F(  4,   195) =   11.20
   Model |  .980169926     4  .245042482           Prob > F      =  0.0000
Residual | 4.26524771    195  .021873065           R-squared     =  0.1869
---------+------------------------------           Adj R-squared =  0.1702
   Total | 5.24541764    199  .026358883           Root MSE      =   .1479

------------------------------------------------------------------------------
   lnSBP |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------+--------------------------------------------------------------------
   woman |   .0035403   .0212026     0.17   0.868    -.0382757    .0453562
         |
agegrp3  |
      1  |   .0715136   .0253373     2.82   0.005     .0215432     .121484
      2  |    .130465   .0280521     4.65   0.000     .0751404    .1857895
         |
 lnBMI25 |   .2898622   .0772432     3.75   0.000     .1375229    .4422015
   _cons |   4.789641   .0224814   213.05   0.000     4.745303    4.833979
------------------------------------------------------------------------------
```

Note 0 is missing: it is base/reference group

Categorical variable : age group 0 reference

$$\ln(sbp)=\alpha_0+\alpha_1\cdot age1+\alpha_2\cdot age2+\alpha_3\cdot woman+\alpha_4\cdot\ln\left(\frac{bmi}{25}\right)+E$$

Adjusted difference between for a person in **age group 1** compared to **age group 0**

```
------------------------------------------------------------------------------
   lnSBP |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------+--------------------------------------------------------------------
   woman |   .0035403   .0212026     0.17   0.868    -.0382757    .0453562
agegrp3  |
      1  |   .0715136   .0253373     2.82   0.005     .0215432     .121484
      2  |    .130465   .0280521     4.65   0.000     .0751404    .1857895
 lnBMI25 |   .2898622   .0772432     3.75   0.000     .1375229    .4422015
   _cons |   4.789641   .0224814   213.05   0.000     4.745303    4.833979
------------------------------------------------------------------------------
```

Adjusted difference between for a person in **age group 2** compared to **age group 0**

Expected value for a **man** in **age group 0** with bmi=25.

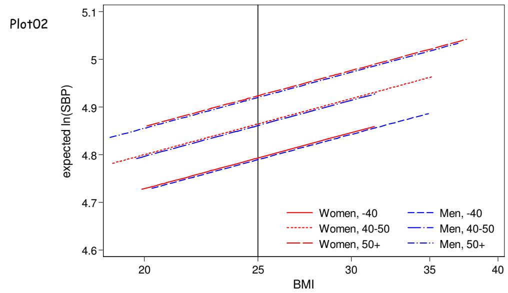
The expected values:

$$age<40: \quad \ln(sbp)=\alpha_0 \qquad\qquad +\alpha_3\cdot woman+\alpha_4\cdot\ln(bmi/25)$$
$$40\le age<50: \quad \ln(sbp)=\alpha_0+\alpha_1 \quad +\alpha_3\cdot woman+\alpha_4\cdot\ln(bmi/25)$$
$$50\le age: \quad \ln(sbp)=\alpha_0+\alpha_2 \quad +\alpha_3\cdot woman+\alpha_4\cdot\ln(bmi/25)$$

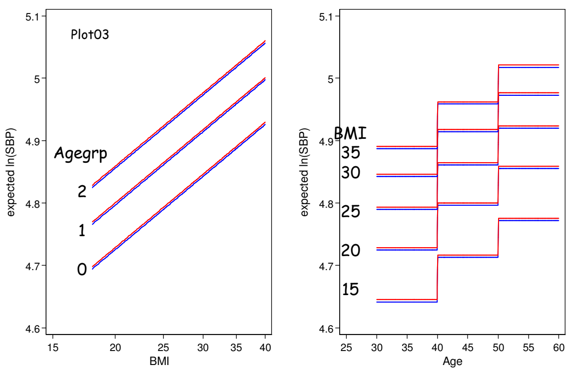
The estimates

lnSBP	Coef.	
woman	.003540	3
1.agegrp3	**.071514**	1
2.agegrp3	**.130465**	2
lnBMI25	.289862	4
_cons	4.789641	0

$$age<40: \quad \ln(sbp)=4.789+ \qquad\qquad 0.004\cdot woman+0.290\cdot\ln(bmi/25)$$
$$40\le age<50: \quad \ln(sbp)=4.789+0.072+0.004\cdot woman+0.290\cdot\ln(bmi/25)$$
$$50\le age: \quad \ln(sbp)=4.789+0.130+0.004\cdot woman+0.290\cdot\ln(bmi/25)$$



Agegroup	Women	Men
0:	$4.793+0.290\cdot\ln(bmi/25)$	$4.790+0.290\cdot\ln(bmi/25)$
1:	$4.865+0.290\cdot\ln(bmi/25)$	$4.861+0.290\cdot\ln(bmi/25)$
2:	$4.924+0.290\cdot\ln(bmi/25)$	$4.920+0.290\cdot\ln(bmi/25)$



Categorical variable : age group 1 reference

$$\ln(sbp)=\gamma_0+\gamma_1\cdot age0+\gamma_2\cdot age2+\gamma_3\cdot woman+\gamma_4\cdot\ln\left(\frac{bmi}{25}\right)+E$$

age0 is one if a person is in age group **0** and zero otherwise
age2 is one if a person is in age group **2** and zero otherwise

The expected $\ln(sbp)$ in the three age groups will be:

$$age<40: \quad \ln(sbp)=\gamma_0+\gamma_1 \quad +\gamma_3\cdot woman+\gamma_4\cdot\ln(bmi/25)$$
$$40\le age<50: \quad \ln(sbp)=\gamma_0 \qquad +\gamma_3\cdot woman+\gamma_4\cdot\ln(bmi/25)$$
$$50\le age: \quad \ln(sbp)=\gamma_0+\gamma_2 \quad +\gamma_3\cdot woman+\gamma_4\cdot\ln(bmi/25)$$

We see that γ_1 is the adjusted difference in $\ln(sbp)$ when comparing a person in the **first group** with one in the **second group**.

And γ_2 is the adjusted difference in $\ln(sbp)$ when comparing a person in the **third group** with one in the **second group**.

Categorical variable : age group 1 reference

$$\ln(sbp)=\gamma_0+\gamma_1\cdot age0+\gamma_2\cdot age2+\gamma_3\cdot woman+\gamma_4\cdot\ln\left(\frac{bmi}{25}\right)+E$$

Finally we see that γ_0 is the expected $\ln(sbp)$ for a man in the **second** (reference) age group, with bmi=25.

Many programs (but regression in SPSS) let you choose the reference group

In Stata 11 this is done like this:

regress `lnSBP woman` **`b1.`**`agegrp3 lnBMI25`

This is categorical with base/reference set to 1

Categorical variable : age group 1 reference

$$\ln(sbp)=\gamma_0+\gamma_1\cdot age0+\gamma_2\cdot age2+\gamma_3\cdot woman+\gamma_4\cdot\ln\left(\frac{bmi}{25}\right)+E$$

```
  Source |       SS       df       MS              Number of obs =     200
---------+------------------------------           F(  4,   195) =   11.20
   Model |  .980169926     4  .245042482           Prob > F      =  0.0000
Residual|  4.26524771   195  .021873065           R-squared     =  0.1869
---------+------------------------------           Adj R-squared =  0.1702
   Total |  5.24541764   199  .026358883           Root MSE      =   .1479

   lnSBP |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------+----------------------------------------------------------------
   woman |   .0035403   .0212026     0.17   0.868    -.0382757    .0453562
         |
agegrp3  |
       0 |  -.0715136   .0253373    -2.82   0.005     -.121484   -.0215432
       2 |   .0589513   .0263496     2.24   0.026     .0069846    .1109181
         |
 lnBMI25 |   .2898622   .0772432     3.75   0.000     .1375229    .4422015
   _cons |   4.861154   .0207406   234.38   0.000      4.82025    4.902059
---------------------------------------------------------------------------
```

Note 1 is missing: it is base/reference group

Categorical variable : age group 1 reference

$$\ln(sbp) = \gamma_0 + \gamma_1 \cdot age0 + \gamma_2 \cdot age2 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

Adjusted difference between for a person in **age group 0** compared to **age group 1**

lnSBP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
woman	.0035403	.0212026	0.17	0.868	-.0382757	.0453562
agegrp3						
0	-.0715136	.0253373	-2.82	0.005	-.121484	-.0215432
2	.0589513	.0263496	2.24	0.026	.0069846	.1109181
lnBMI25	.2898622	.0772432	3.75	0.000	.1375229	.4422015
_cons	4.861154	.0207406	234.38	0.000	4.82025	4.902059

Adjusted difference between for a person in **age group 2** compared to **age group 1**

Expected value for a man in **age group 1** with *bmi*=25.

Categorical variable: Comparing two parameterizations

$$\ln(sbp) = \alpha_0 + \alpha_1 \cdot age1 + \alpha_2 \cdot age2 + \alpha_3 \cdot woman + \alpha_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

$$\ln(sbp) = \gamma_0 + \gamma_1 \cdot age0 + \gamma_2 \cdot age2 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

age group 0: $\alpha_0 = \gamma_0 + \gamma_1$

age group 1: $\alpha_0 + \alpha_1 = \gamma_0$

age group 2: $\alpha_0 + \alpha_2 = \gamma_0 + \gamma_2$

$\alpha_0 = \gamma_0 + \gamma_1$	$\gamma_0 = \alpha_0 + \alpha_1$
$\alpha_1 = -\gamma_1$	$\gamma_1 = -\alpha_1$
$\alpha_2 = \gamma_2 - \gamma_1$	$\gamma_2 = \alpha_2 - \alpha_1$
$\alpha_3 = \gamma_3$	$\gamma_3 = \alpha_3$
$\alpha_4 = \gamma_4$	$\gamma_4 = \alpha_4$

Categorical variable: Comparing two parameterizations

The estimates:

age group 0 reference

Root MSE = .1479

lnSBP	Coef.	[95% CI]	
woman	.0035	-.0382	.0453
agegrp3			
1	.0715	.0215	.1214
2	.1304	.0751	.1857
lnBMI25	.2898	.1375	.4422
_cons	4.7896	4.745	4.833

age group 1 reference

Root MSE = .1479

lnSBP	Coef.	[95% CI]	
woman	.0035	-.0382	.0453
agegrp3			
0	-.0715	-.1214	-.0215
2	.0589	.0069	.1109
lnBMI25	.2898	.1375	.4422
_cons	4.8611	4.820	4.902

Note, the estimates fulfil the same equations.

The interpretation of the " **agegrp3 2** line" and " **_cons** line" are altered!!!!!!!!!

Always remember: what is the reference group!

Categorical variable : age group 1 reference

lnSBP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
woman	.0035403	.0212026	0.17	0.868	-.0382757	.0453562
agegrp3						
0	-.0715136	.0253373	-2.82	0.005	-.121484	-.0215432
2	.0589513	.0263496	2.24	0.026	.0069846	.1109181
lnBMI25	.2898622	.0772432	3.75	0.000	.1375229	.4422015
_cons	4.861154	.0207406	234.38	0.000	4.82025	4.902059

Two test:

One testing no difference between age group **0** and **1**.
One testing no difference between age group **2** and **1**.

Can we get **one test** testing no difference between age groups?

An F-test in Stata: *testparm* `i.agegrp`

```
 ( 1)  0.agegrp3 = 0
 ( 2)  2.agegrp3 = 0
       F(  2,   195) =   10.93
            Prob > F =    0.0000
```

Highly significant

Interactions/effectmodification

$$\ln(sbp) = \gamma_0 + \gamma_1 \cdot age0 + \gamma_2 \cdot age2 + \gamma_3 \cdot woman + \gamma_4 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

One of the **central** assumptions was "no effect modification".

E.g. in the model above the "effect" of **age**, **sex** and **bmi** did not depend on the value of each other.

One can **introduce** effect modification between a categorical variable and another variable.

Here we first will look at **agegr3** and **lnBMI25**.

The effect modification will be that the coefficient to **lnBMI25** depend on **age group**.

That is, we will allow **different effect** of bmi in the **different age** groups.

Interactions/effectmodification

$$\ln(sbp) = \omega_0 + \omega_1 \cdot age0 + \omega_2 \cdot age2 + \omega_3 \cdot woman$$
$$+\omega_4 \cdot \ln\left(\frac{bmi}{25}\right) + \omega_5 \cdot age0 \cdot \ln\left(\frac{bmi}{25}\right) + \omega_6 \cdot age2 \cdot \ln\left(\frac{bmi}{25}\right) + E$$

$age \le 40$: $\ln(sbp) = (\omega_0 + \omega_1) + (\omega_4 + \omega_5) \cdot \ln\left(\frac{bmi}{25}\right) + \omega_3 \cdot woman$

$40 < age \le 50$: $\ln(sbp) = \omega_0 + \omega_4 \cdot \ln\left(\frac{bmi}{25}\right) + \omega_3 \cdot woman$

$50 < age$: $\ln(sbp) = (\omega_0 + \omega_2) + (\omega_4 + \omega_6) \cdot \ln\left(\frac{bmi}{25}\right) + \omega_3 \cdot woman$

ω_1 is the **difference** between the constant for age **group 0** and **reference** group.
ω_5 is the **difference** between the coefficient to **lnBMI25** for age **group 0** and **reference** group.

Interactions/effectmodification

```
regress lnSBP woman b1.agegrp3##c.lnBMI25
  Source |       SS       df       MS              Number of obs =     200
---------+------------------------------           F(  6,   193) =    7.53
   Model |  .994860827     6  .165810138           Prob > F      =  0.0000
Residual |  4.25055681   193   .02202361           R-squared     =  0.1897
---------+------------------------------           Adj R-squared =  0.1645
   Total |  5.24541764   199  .026358883           Root MSE      =   .1484

   lnSBP |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------+----------------------------------------------------------------
   woman |   .0076438   .0219199     0.35   0.728    -.0355896    .0508772
         |
 agegrp3 |
      0  |  -.0708045   .0261198    -2.71   0.007    -.1223213   -.0192877
      2  |   .0631082   .0270342     2.33   0.021     .0097877    .1164287
         |
 lnBMI25 |   .3155479   .1222905     2.58   0.011     .0743505    .5567453
         |
 agegrp3#|
c.lnBMI25|
      0  |   .0429736   .1912373     0.22   0.822    -.3342099     .420157
      2  |  -.1165375   .1855477    -0.63   0.531    -.4824991    .2494242
         |
   _cons |   4.859743   .0214122   226.96   0.000     4.817511    4.901975
--------------------------------------------------------------------------
```


Interactions/effect modification

Ref: constant and 'slope' in reference group

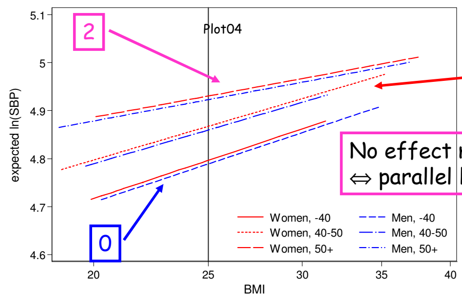
0: difference in constant and slope compared to reference

2: difference in constant and slope compared to reference

```
   lnSBP |     Coef.    Std. E.       t    P>|t|     [95% Conf. Interval]
---------+----------------------------------------------------------------
   woman |  .0076438   .0219199    0.35   0.728    -.0355896    .0508772
 agegrp3 |
      0  | -.0708045   .0261198   -2.71   0.007    -.1223213   -.0192877
      2  |  .0631082   .0270342    2.33   0.021     .0097877    .1164287
 lnBMI25 |  .3155479   .1222905    2.58   0.011     .0743505    .5567453   Ref
 agegrp3#|
c.lnBMI25|
      0  |  .0429736   .1912373    0.22   0.822    -.3342099     .420157
      2  | -.1165375   .1855477   -0.63   0.531    -.4824991    .2494242
   _cons |  4.859743   .0214122  226.96   0.000     4.817511    4.901975   Ref
--------------------------------------------------------------------------
```

Note the larger standard errors

Based on the estimates one can find the six "dose-response" curves:



No effect modification ⇔ parallel lines.

Agegroup	Women	Men
0:	$4.797+0.359\cdot\ln(bmi/25)$	$4.789+0.359\cdot\ln(bmi/25)$
1:	$4.867+0.316\cdot\ln(bmi/25)$	$4.860+0.316\cdot\ln(bmi/25)$
2:	$4.930+0.199\cdot\ln(bmi/25)$	$4.923+0.199\cdot\ln(bmi/25)$

Interactions/effect modification

```
   lnSBP |     Coef.    Std. E.       t    P>|t|     [95% Conf. Interval]
---------+----------------------------------------------------------------
   woman |  .0076438   .0219199    0.35   0.728    -.0355896    .0508772
 agegrp3 |
      0  | -.0708045   .0261198   -2.71   0.007    -.1223213   -.0192877
      2  |  .0631082   .0270342    2.33   0.021     .0097877    .1164287
 lnBMI25 |  .3155479   .1222905    2.58   0.011     .0743505    .5567453
 agegrp3#|
c.lnBMI25|
      0  |  .0429736   .1912373    0.22   0.822    -.3342099     .420157
      2  | -.1165375   .1855477   -0.63   0.531    -.4824991    .2494242
   _cons |  4.859743   .0214122  226.96   0.000     4.817511    4.901975
```

Two tests:

One testing differences between "slope" in age group **0** and **1**.

One testing differences between "slope" in age group **2** and **1**.

One **test** testing no difference between age groups!

A F-test in Stata: *testparm* i.agegrp3#c.lnBMI25

```
 ( 1)  0.agegrp3#c.lnBMI25 = 0
 ( 2)  2.agegrp3#c.lnBMI25 = 0
       F(  2,   193) =    0.33
            Prob > F =    0.7168
```

Non-significant

Interactions/effect modification

The test of no interaction was non-significant.

But look at the **confidence interval** for the difference in slope for between age group **2** and group **1** !

```
   lnSBP |     Coef.    Std. E.       t    P>|t|     [95% Conf. Interval]
---------+----------------------------------------------------------------
   woman |  .0076438   .0219199    0.35   0.728    -.0355896    .0508772
 agegrp3 |
      0  | -.0708045   .0261198   -2.71   0.007    -.1223213   -.0192877
      2  |  .0631082   .0270342    2.33   0.021     .0097877    .1164287
 lnBMI25 |  .3155479   .1222905    2.58   0.011     .0743505    .5567453
 agegrp3#|
c.lnBMI25|
      0  |  .0429736   .1912373    0.22   0.822    -.3342099     .420157
      2  | -.1165375   .1855477   -0.63   0.531    -.4824991    .2494242
   _cons |  4.859743   .0214122  226.96   0.000     4.817511    4.901975
```

It is very wide!!! We know very little about this difference!

The test for no interaction has **very low power**!!!

The data have very **little information** on whether there is effect modification.

Interaction between age group and sex

regress lnSBP lnBMI25 **b1.agegrp3##b1.sex** Male sex==1

```
       lnSBP |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnBMI25 |   .2265018   .0774898     2.92   0.004     .0736662    .3793374
             |
     agegrp3 |
          0  |  -.0426734   .0377096    -1.13   0.259    -.1170493    .0317025
          2  |  -.0025412   .0365457    -0.07   0.945    -.0746215    .0695391
             |
       2.sex |  -.0210869   .0322283    -0.65   0.514    -.0846518     .042478
             |
 agegrp3#sex |
        0 2  |  -.0548967   .0501668    -1.09   0.275    -.1538422    .0440488
        2 2  |    .133379   .0501308     2.66   0.008     .0345043    .2322536
             |
       _cons |   4.873442   .0251767   193.57   0.000     4.823786    4.923099
------------------------------------------------------------------------------

. testparm i.agegrp3#i.sex

 ( 1)  0.agegrp3#2.sex = 0
 ( 2)  2.agegrp3#2.sex = 0

       F(  2,   193) =    6.26
            Prob > F =    0.0023
```

Highly significant

Interaction between age group and sex

Differences between age groups among men are small

lnSBP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnBMI25	.2265018	.0774898	2.92	0.004	.0736662	.3793374
agegrp3						
0	-.0426734	.0377096	-1.13	0.259	-.1170493	.0317025
2	-.0025412	.0365457	-0.07	0.945	-.0746215	.0695391
2.sex	-.0210869	.0322283	-0.65	0.514	-.0846518	.042478
agegrp3#sex						
0 2	-.0548967	.0501668	-1.09	0.275	-.1538422	.0440488
2 2	.133379	.0501308	2.66	0.008	.0345043	.2322536
_cons	4.873442	.0251767	193.57	0.000	4.823786	4.923099

Women age group 1: $4.873-0.021=4.852$

Women age group 0: $4.873-0.021-0.042-0.055=4.755$ (-0.097)

Women age group 2: $4.873-0.021-0.003+0.133=4.982$ (0.130)

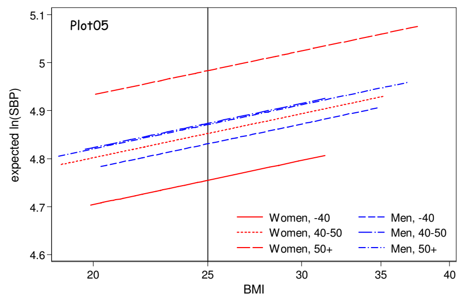
Large differences in the age groups among women.

Interaction between age group and sex

Using women as reference: b2.sex

Large differences between age groups among women

lnSBP	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnBMI25	.2265018	.0774898	2.92	0.004	.0736662	.3793374
agegrp3						
0	-.0975701	.0328469	-2.97	0.003	-.162355	-.0327852
2	.1308378	.0354804	3.69	0.000	.0608587	.2008168
1.sex	.0210869	.0322283	0.65	0.514	-.042478	.0846518
agegrp3#sex						
0 1	.0548967	.0501668	1.09	0.275	-.0440488	.1538422
2 1	-.133379	.0501308	-2.66	0.008	-.2322536	-.0345043
_cons	4.852355	.0205502	236.12	0.000	4.811824	4.892887



Agegroup	Women	Men
0:	$4.755+0.227\cdot\ln(bmi/25)$	$4.831+0.227\cdot\ln(bmi/25)$
1:	$4.852+0.227\cdot\ln(bmi/25)$	$4.873+0.227\cdot\ln(bmi/25)$
2:	$4.982+0.227\cdot\ln(bmi/25)$	$4.871+0.227\cdot\ln(bmi/25)$