Ph.D. COURSE IN BIOSTATISTICS DAY 1

Interpretation of **evidence** is an important part of medical research. Evidence is often in the form of **numerical data**.

Statistics: Methods for collection, analysis and evaluation of data

Statistics

Descriptive statistics

Tables and plots that highlights important aspects of the data.

Inferential statistics

Analysis and evaluation of evidence in the form of numerical data. The purpose of the statistical analysis is to extract relevant information about a (scientific) problem and to draw conclusions about a population from a sample of individuals.

Statistics also include aspects of the ***design*** of experiments and observational studies

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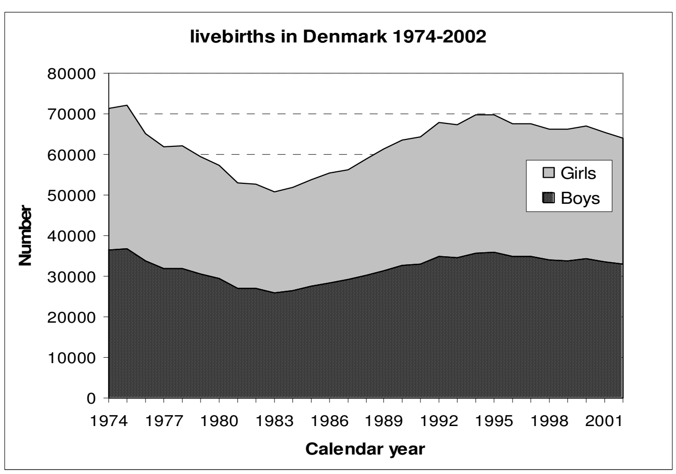
A simple example

In Denmark the number of live births per year has varied considerably over the last 20-30 years



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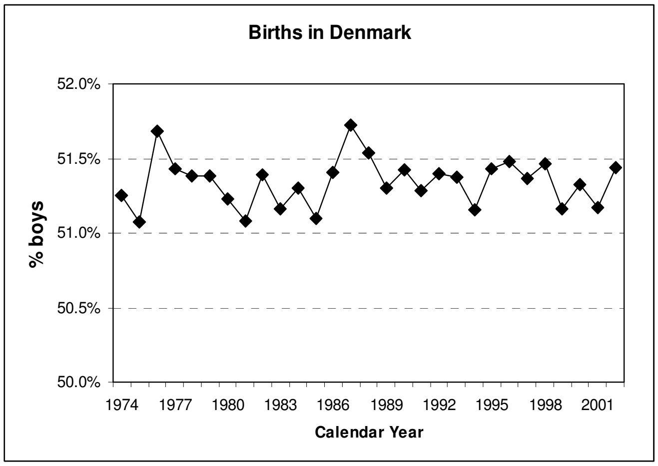
In the same period the percentage of boys has been very stable.



From 1974 to 2002 the percentage of boys varied between 51.1% and 51.7% with an average of 51.34%

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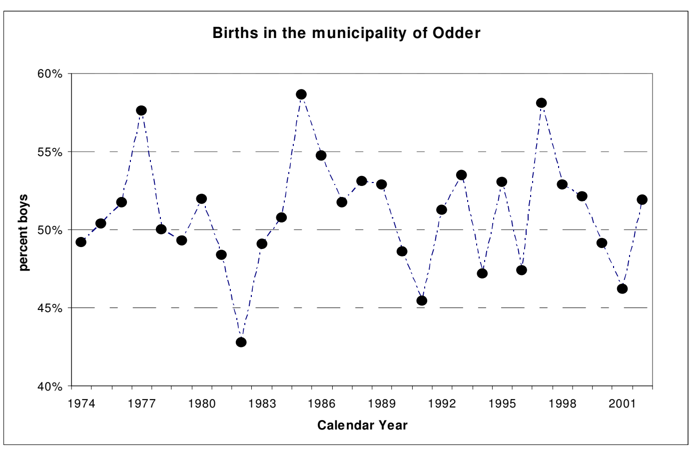
In smaller communities the percentage of boys varied much more.
Often the number of girls exceeded the number of boys:



Does these data suggest that the percentage of boys born in Odder differs from the national figure?
Is the variation from year to year larger than what is expected?

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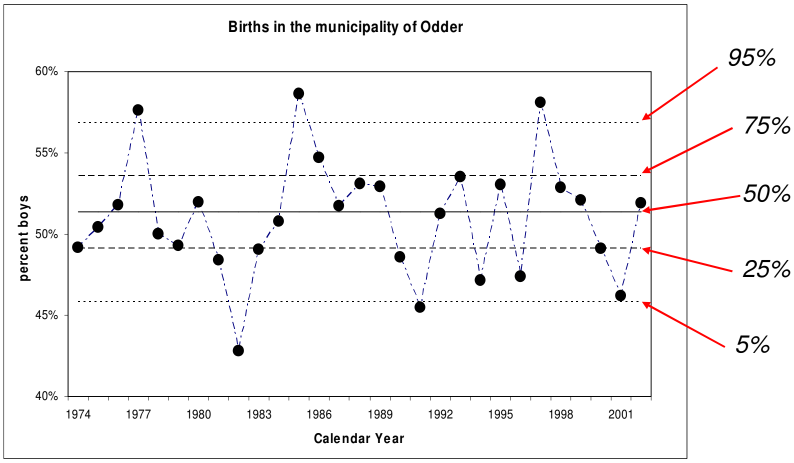
The number of births in Odder varied between 180 and 256 in the period with an average of 224.



The lines show so-called 5, 25, 50, 75 and 95-percentiles of the distribution of a proportion based on a sample of size 224 when the true proportion is 51.3%

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For a (random) sample of size 224 statistical theory predicts that

- 50% of the proportions fall above the full line and 50% below
- 50% of the proportions fall between the two dashed lines
- 90% of the proportions fall between the two dotted lines

We cannot expect to get exactly these percentages in every sample.

Statistical methods allows us to describe the data and evaluate if the observations are consistent with the expectations.

In this example: information about the total population is known, so we were able to describe the variation in the samples from these results.

Usually: No data on population level. Instead one or several samples are available. The population may be rather unspecific, e.g. ”similar patients”

Typical problems: compare samples, describe associations

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Basic components of a statistical analysis

Specification of a ***statistical model***.
The model gives a formal description of the systematic differences and the randomness in the data

Estimation of population characteristics.
These are often parameters of the ***statistical model***

Validation of the ***statistical model***.
The relevance of the statistical analysis depends on the validity of the assumptions

Testing hypotheses about the ***model parameters***.
The hypotheses reflects the (scientific) questions that led to the data collection

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Statistics – some key aspects

Common sense
Inductive inference
Study of variation
Methods for data reduction
Models
Quantification of uncertainty
Mathematics
Computer programs

Inductive inference
Methods for drawing conclusion about a ***population*** of individuals from a ***sample*** of specific individuals

Variation
Biological data are usually subject to large variation:
Which aspects of the variation in the data represent ***systematic*** differences?
Which aspects of the variation in the data are ***random*** fluctuations?

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Variation (continued)

Studies are often carried out to assess the size of systematic effects or to correct for some (known) systematic effects when evaluating the size of other sytematic effects.

Many sources of random variation, e.g.

- Measurement error
- Biological variation: Intra- and inter-individual variation
- Observer variation

Data reduction

How do we describe a large data set with a few relevant numbers without loosing important information?

Statistical models – Quantification of uncertainty - Mathematics

An idealized probabilistic description of the proces that generates the data. The model includes some ***parameters*** which describe systematic and random variation.

The statistical analysis gives estimates of the parameters and limits of uncertaincy for the estimates.

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Computer programs

Statistical analyses often involve large amount of computations.

Simple calculations can be done by pocket calculators and spreadsheet, but special-purpose software are usually convenient.

In this course the program **Stata** version 8 is used for all calculations. For an easy introduction to **Stata** see:

Svend Juhl. Introduction to Stata 8. Aarhus 2003.

A pdf-file can be downloaded from

http://www.biostat.au.dk/teaching/software/

In Stata 8 statistical calculations and plots can be specified either by use of pull-down menus or by use of commands.

Menus are especially convenient for beginners and occasional users, but once the basic commands are learned the command language provide a much faster way to interact with the program, and a series of commands can be saved for documantation or later use.

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A typical Stata session involves

Starting Stata: Double-click on the Stata icon

Defining a log-file to contain the results

```
log using "e:\kurser\f2004\outfile.log
log using "e:\kurser\f2004\outfile.log , text
```

reading the data

```
use "e:\kurser\f2004\mydata.dta"
```

a series of commands that perform the desired analyses

saving the results

```
log close
```

saving the new data, if data have been modified

```
save "e:\kurser\f2004\mynewdata.dta
```

A series of commands can be created and saved in a command file, `mycommand.do`, and run in batch mode from within Stata

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STATA – basic commands for data manipulation

Example 1

The file ***skejby-cohort.dta*** contains information on the mother and the newborn for all deliveries in the maternity ward at Skejby Hospital in period 1993-95.

The information in the file is stored in 8 variables. To get the data into Stata and list the first 5 records

```
cd E:\kurser\f2004\
log using outfile01.log , text
use skejby-cohort
list in 1/5
```

	bweight	gestage	mtobacco	cigarets	bsex	mage	parity	date
1.	4000	41	.	.	girl	.	0	30793
2.	2640	36	.	.	girl	.	0	141095
3.	3000	39	smoker	10	girl	41	2	60995
4.	3330	41	.	.	boy	39	2	100694
5.	3700	39	nonsmoker	0	girl	39	1	10495

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To list only records satisfying a specified condition use

```
list bweight bsex mage if mage>45
       +-----------------------+
       | bweight   bsex   mage |
       |-----------------------|
    1. |    4000   girl      . |
    2. |    2640   girl      . |
 1008. |    3745    boy      . |
 2079. |    3100    boy     46 |
       +-----------------------+
```

Note that missing values are included since in Stata a missing value is represented by a (very) large number

Other uses of ***list***

```
list mage in 1/100 if parity==0
list if bsex==. , clean compress
```

compact format with no lines

missing value

Notes

- A logical "equal to" is written as ==
- Options are place after a comma. A full list of options are available in the help-menu: Help→Stata Command…→write name of the command in field→OK or using the keyboard: Alt-h-o

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Generating new variable and changing existing variables

New variables are defined using the command ***generate***.
Existing variables are changed with the command ***replace***

Example

```
generate day=int(date/10000)
generate mon=int((date-10000*day)/100)
generate year=1900+date-10000*day-100*mon
generate primi=(parity==0) if parity<.
list day mon year date parity primi in 1/2 , clean
```

Output:

```
      day   mon   year     date   parity      primi
 1.     3     7   1993    30793        0          1
 2.    14    10   1995   141095        0          1
```

Variables can be labeled to further explain the contents

```
label variable day "day of birth"
label variable mon "month of birth"
label variable year "year of birth"
label variable primi "first child"
```

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Labels may also be added to the categories of a categorical variables

```
label define primilab 0 "multipari" 1 "primipari"
label values primi primilab
list parity primi in 1/2 , clean
```

```
        parity       primi
   1.        0   primipari
   2.        0   primipari
```

Date variables are a special type of variables. Dates are represented as days since January 1 1960, but can easily be shown in a more useful format

```
generate bdate=mdy(mon,day,year)
list day mon year date bdate in 1 , clean
format bdate %d
list date bdate in 1 , clean
```

number of days since 1.1.1960

```
        day   mon    year    date   bdate
  1.      3     7    1993   30793   12237
--------
         date       bdate
  1.    30793   03jul1993
```

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New variables can also be defined using the command ***recode.***
The variable **primi** generated above can alternatively be defined as

```
recode parity (0=1)(.=.)(else=0) , generate(primi)
```

Note: the option generate ensures that the information is placed in a new variable. If omitted the result is placed in the old variable and the original contents is loss.

Adding a column with record numbers to the file

```
generate recno=_n
```

Sorting the data according to a variable

```
sort mage
```

Renaming and reordering of variables

```
rename bsex sex
order recno bdate
```

Any variable not mentioned follows the variables mentioned

A short description of a variable, including labels (if any) and format

```
describe mage mtobacco
```

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Selection of records and variables

To keep only data on births from 1993

```
keep if year==1993
```

To drop records for mothers younger than 20

```
drop if mage < 20
```

To keep variables from *bweight* to *cigarets* and *parity*

```
keep bweight-cigarets parity
```

To drop the variables *mtobacco* and *cigarets*

```
drop mtobacco cigarets
```

To keep 15% random sample of the data in memory

```
sample 15
```

To obtain a random sample of size 200 records (other records are dropped)

```
sample 200, count
```

To recover all data the file must be re-read into memory with `use`

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SUMMARIZING DATA - DESCRIPTIVE STATISTICS

Main data types

- Qualitative or categorical data
- Quantitative data – two subtypes: discrete and continuous

The data contain information on different, qualitative or quantitative, aspects of the individuals/objects in the sample.
These aspects are usually called ***variables***

Qualitative variables: Each individual/object falls in a class or category
The categories may be ***ordered*** (e.g. low, moderate, high) or ***unordered*** (e.g. boy, girl). In the data file the categories are assigned (arbitrary) numbers, but lables may be used to clarify the meaning.

Examples of qualitative variables in the file *skejby-cohort.dta*

bsex "sex of child"
the variable has categories *"boy"* and *"girl"*

mtobacco "smoking habits of mother"
the variable has categories "*smoker*" and "*nonsmoker*"

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The categories of a qualitative variable, their numerical codes and labels can be displayed using the command ***codebook***

Example

The command ***codebook* bsex** gives the following output

```
-------------------------------------------------------------------
bsex
sex of child
-------------------------------------------------------------------
                  type:  numeric (float)
                 label:  sexlab

                 range:  [1,2]                  units:  1
         unique values:  2                  missing .:  4/12955

            tabulation:  Freq.   Numeric  Label
                          6674         1  boy
                          6277         2  girl
                             4         .
```

Frequencies, and ***relative frequencies*** of the different categories can also be obtained with the commands ***tabulate*** or ***tab1***.

Missing values are ignored unless the option ***missing*** is added.

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Examples

tabu* bsex *, missing

tabu* bsex year *, col nofreq nokey

Output

```
     sex of |
      child |      Freq.     Percent        Cum.
------------+-----------------------------------
        boy |      6,674       51.52       51.52
       girl |      6,277       48.45       99.97
          . |          4        0.03      100.00
------------+-----------------------------------
      Total |     12,955      100.00

     sex of |          year of birth
      child |      1993       1994       1995 |     Total
-----------+--------------------------------+----------
        boy |     51.37      51.80      51.43 |     51.53
       girl |     48.63      48.20      48.57 |     48.47
-----------+--------------------------------+----------
      Total |    100.00     100.00     100.00 |    100.00
```

Several other options are also available. Using ***if*** and ***in*** tabulations can be restricted to subsets of the data, e.g.

***tabu* bsex year *if* mage>35**

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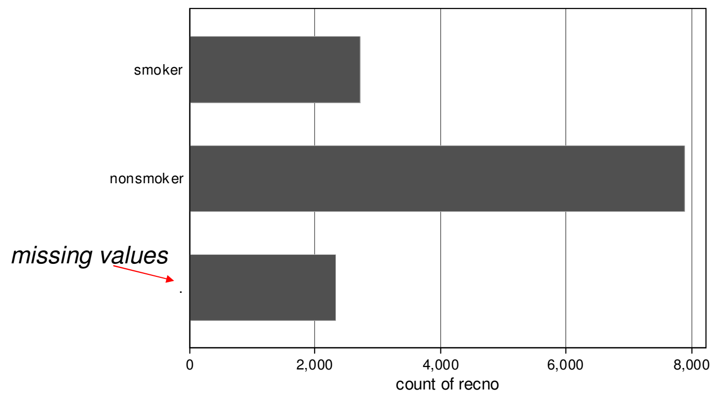
The tabulations can be displayed as bar charts or pie charts with the commands, e.g.

```
graph hbar (count) recno, over(mtobacco) missing
graph bar (count) recno, over(mtobacco) missing
graph pie , over(mtobacco) missing
```

The option ***missing*** includes a category for missing values.
The first command produces the following graph:



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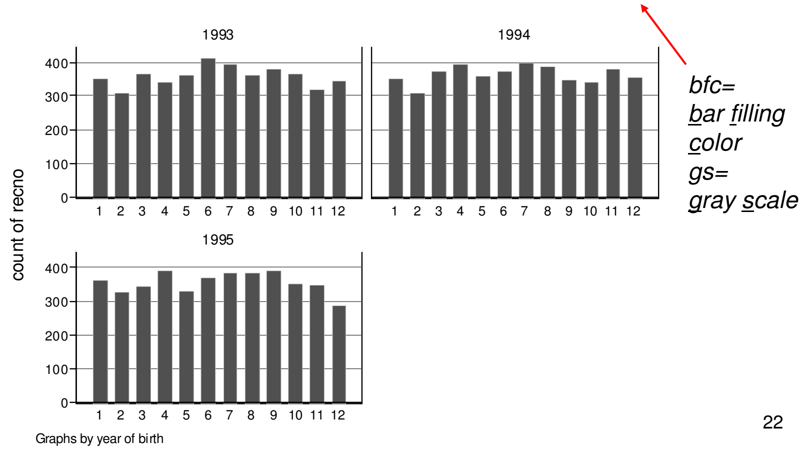
Separate plots for each value of another variable may easily be obtained

Example: Plots of the number of births for each months of the year for each of the years 1993, 1994 and 1995 produced by

```
graph bar (count) recno ,
        over(mon) by(year) bar(1,bfc(gs5))
```



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SUMMARIZING QUANTITATIVE VARIABLES

Quantitative variables contain numerical information. Two types
Discrete variables : only integer values are possible.
Example: *parity*.

Continuous variables: Measurements can in principle be any number in a range (due to rounding not all numbers may occur in practice)
Examples: *bweight, mage*

For quantitative variables a table of frequencies or relative frequencies is less useful.

When summarizing the values of a quantitative variable focus is usually on

What is a typical value?
How much variation is there in the data?

For each question several data summaries, so-called ***statistics***, are available

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Typical value
Data: a sample of *n* observations $y_1, y_2, \ldots, y_n$

The ***sample mean*** is the average of the observations

$$\bar{y} = (y_1 + \cdots + y_n)/n = \frac{1}{n}\sum_{i=1}^{n} y_i$$

The ***sample median:*** the value which separates the smallest 50% from the largest 50%

Percentiles (or ***quantiles***)
5-percentile: The value for which 5% of the observations are smaller than this value
10-percentile: The value for which 10% of the observations are smaller than this value
etc.

25-percentile is called the ***lower quartile***, 50-percentile is the median, 75-percentile is called the ***upper quartile***

The ***quartiles*** divide the observations in four groups of the same size.

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Describing variation in the data

The average deviation from the sample mean is always 0, so this is not a useful measure of the variation in the data.
One could instead consider the average absolute deviation from the sample mean, but this number is mathematically less attractive.
The average squared deviation is usually preferred

Sample variance

$$s^2 = \frac{1}{n-1}\left[\left(y_1 - \bar{y}\right)^2 + \cdots + \left(y_n - \bar{y}\right)^2\right] = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2$$

Sample standard deviation is the square root of the sample variance and is measured in the same units as the observations

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}$$

Range: The largest observation minus the smallest observation

Interquartile range: The upper quartile minus the lower quartile

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Summarizing quantitative variables with Stata

```
summarize bweight gestage mage
```

Output:

```
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
     bweight |     12955    3517.842    584.7701        520       5880
     gestage |     12851    39.52751    1.883722         24         44
        mage |     12952    28.89554    4.828905         15         46
```

The option ***detail*** gives additional information

```
summarize bweight , detail
```

Output:

```
birth weight
-------------------------------------------------------------
      Percentiles      Smallest
 1%         1605            520
 5%         2580            580
10%         2850            600       Obs               12955
25%         3200            610       Sum of Wgt.       12955

50%         3540                      Mean           3517.842
                        Largest       Std. Dev.      584.7701
75%         3900           5620
90%         4200           5640       Variance         341956
95%         4400           5750       Skewness      -.6593171
99%         4800           5880       Kurtosis       5.238386
```

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Summary statistics of a quantitative variable for each category of a qualitative variable

```
bysort year: summarize bweight
```

Alternative command for obtaining selected summary statistics

```
tabstat bweight gestage mage,
          stat(n mean med p5 p95 sd range iqr)
```

Output:

```
   stats |   bweight   gestage      mage
---------+------------------------------
       N |     12955     12851     12952
    mean |  3517.842  39.52751  28.89554
     p50 |      3540        40        29
      p5 |      2580        36        21
     p95 |      4400        42        37
      sd |  584.7701  1.883722  4.828905
   range |      5360        20        31
     iqr |       700         2         6
----------------------------------------
```

Options include 25 different summary statistics. See help on `tabstat` for details. Also a `by()` option is allowed, e.g.

```
tabstat bweight , stat(n mean sd) by(bsex)
```

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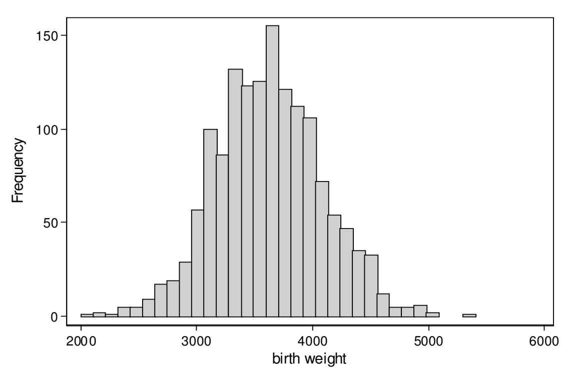
DISPLAYING QUANTITATIVE DATA - GRAPHS

Useful plots for describing a set of observations include ***histograms***, ***cumulative distribution functions*** and ***Box-plots***.

The command

```
histogram bweight if year==1993 & gestage==40,freq
```

gives the following plot



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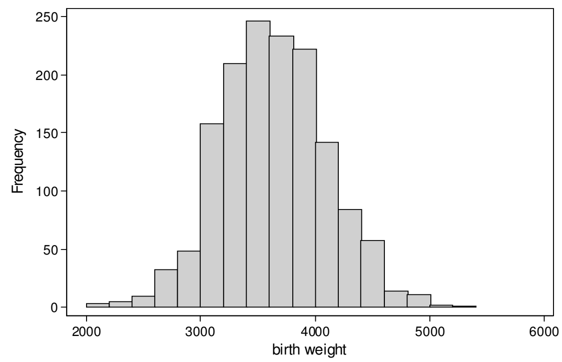
The user may specify options to define the number of categories – called bins – the starting point and the width of the categories. Otherwise Stata finds suitable values.
In the plot above Stata chose bin=31, start=2000, width=106.45161
The command

```
histogram bweight if year==1993 & gestage==40 ,
          freq start(2000) width(200)
```

gives the following plot



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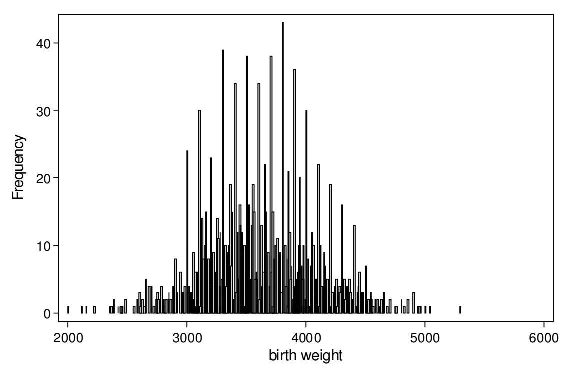
The option ***freq*** gives frequency (counts) on the y-axis.
If omitted the density is shown (i.e. total *area*=1).

"Uncategorized" data are plotted with the option ***discrete***, e.g.

```
histogram bweight if year==1993 & gestage==40 ,
    freq discrete
```



Some birth weights are more popular than others!

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Interpretation of standard deviations

The distribution of birth weight for a given gestational age looks fairly ***symmetric***. For such distributions we have that

Approximately 67% of the observations fall in the interval from $\bar{y}-s$ to $\bar{y}+s$

Approximately 95% of the observations fall in the interval from $\bar{y}-2s$ to $\bar{y}+2s$

Example
The *skejby-cohort.dta* includes 1477 birth weights in week 40 in 1993.

For this sample we have an average birth weight of 3622.7 gram and the standard deviation is 463.0 gram.

The interval *mean±sd* becomes [3159.7,4085.7] and contains 68.9% (1018 out of 1477) observations.

The interval *mean±2sd* becomes [2696.7,4548.7] and contains 95.5% (1411 out of 1477) observations.

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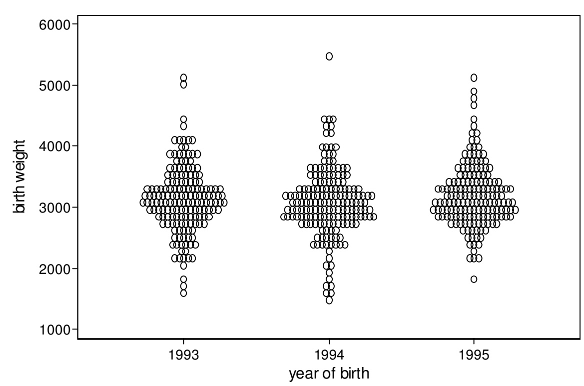
For small data sets ***dot plots*** provide a useful alternative to histograms, especially when comparing two or several groups. e.g.

```
dotplot bweight if gestage==37 ,
        center by(year) mcolor(black)
```

dots are centered — *marker color*



Several additional options are available.

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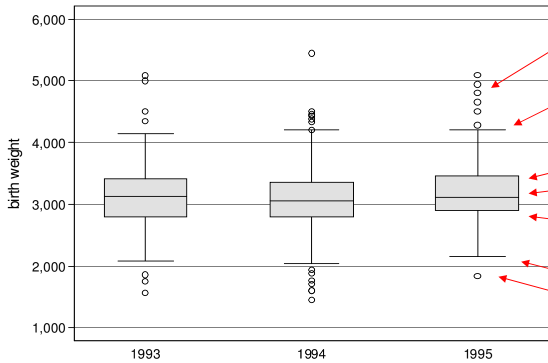
Box plots, also called box-and-whiskers plots, are related to dot plots, but can also be used with larger data sets. Box plots shows quartiles and medians as a box, lines indicate position of data outside the quartiles, and individual outliers are shown.

bar line color
marker color

```
graph box bweight if gestage==37 ,
      over(year) bar(1,blc(black)) m(1,mco(black))
```



Outliers

*Upper adjacent value = largest obs. smaller than upper quatile plus 1.5*IQR*

Upper quartile
Median
Lower Quartile

Defined similarly

Note: Several versions are in use – definitions of specific details differ!

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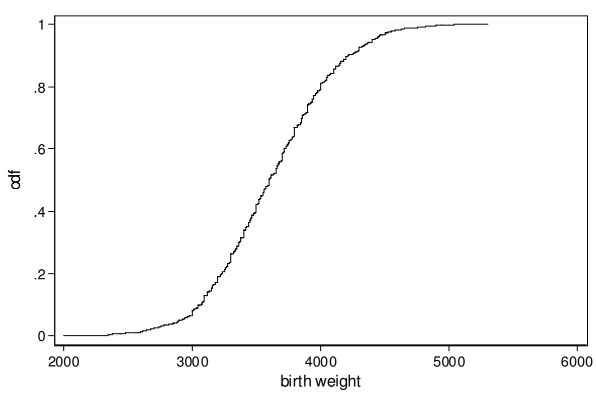
The ***cumulative distribution function*** shows the cumulative frequencies of the observations i.e. the proportion of observations smaller than or equal to *x* plotted against *x*.

```
cumul bweight if year==1993 & gestage==40 ,
         generate(cdf) equal
line cdf bweight if year==1993 & gestage==40 ,
         sort connect(J)
```

connect points with a step function



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Descriptive statistics – from the sample to the population

The descriptive methods are mainly useful for an initial phase of the analysis where the purpose is to understand the main features of the data and in the final stages of the analysis where the purpose is to communicate the main findings

The ***statistical summaries*** of the collected data are usually interpreted as ***estimates of the similar quantities*** in the population from which the sample are drawn.

The population may be well-defined, e.g. all birth in Denmark in a given year, but often the population is rather vaguely defined, e.g. similar patients treated in the future.

Moreover, due to selection, missing values and non-response the population that the sample represents may not be identical to the population for which we want to draw conclusion.

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Examples

1. Assuming that births in Odder in 1995 can be considered as a random sample of births in Denmark (in 1995) the relative frequency, or ***proportion***, of boys observed there is interpreted as ***an estimate*** of the ***probability*** of a randomly selected (singleton) pregnancy in Denmark results in boy.

2. For babies born in week 40 and included in the Skejby cohort the 10 percentile of the birth weight distribution was 3070 gram. Thus, the ***estimated probability*** of a birth weight under 3070 gram for a baby born in week 40 is 10% (for babies born in Denmark in the mid-90's).

3. The ***average birth weight*** for children born in week 38 and included in the Skejby cohort is 3274 gram. Thus, the ***expected birth weight*** in week 38 is estimated to 3274 gram (for babies born in Denmark in the mid-90's).

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From relative frequencies to probabilities

For ***categorical variables*** and ***discrete variables*** we used a ***table*** to display the relative frequency of each possible outcome in the sample.

If the sample size increases to infinity (the sample becomes the total population) these relative frequencies are interpreted as probabilities showing the ***theoretical distribution*** of the random variable.

The terminology ***" a random variable"*** is used to stress that the values of a variable vary in a random fashion among individuals in a population.

The probabilities describe the outcome for a randomly selected individual from the population or the distribution of the random variable.

For ***continuous variables*** such tables are not feasible since the outcome may take any value in a given range and an infinite number of outcomes are therefore possible. The theoretical distribution is therefore usually described by a ***cumulative distribution function*** or a ***probability density function.***

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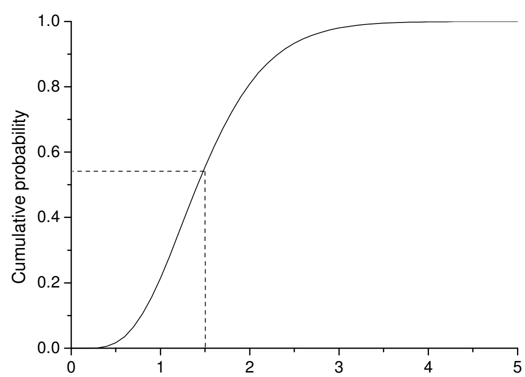
For a random variable *X* with cumulative distribution function *F* we have:

The value of the cumulative distribution function at *a*, *F*(*a*), describes the probability of an outcome less than or equal to *a*.

$F(a) = P(X \leq a) =$ Outcome of *X* is less than or equal to *a*



Example

$F(1.5) = 0.54$

The probability of a value less than 1.5 is equal to 54%

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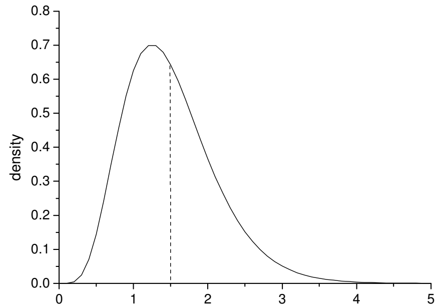
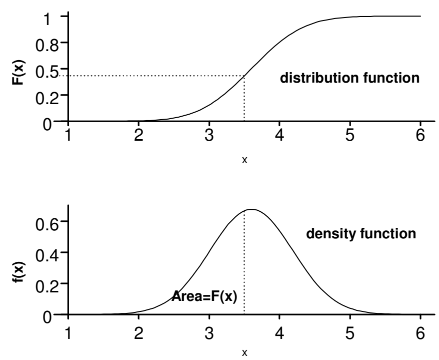
The probability density function, or density function, is the theoretical counterpart of a histogram scaled such that the area under the curve is 1



Relation between density function f and cumulative distribution function F

The ***area*** under the density function up to a is equal to *F*(*a*), the probability that *X* is less than or equal to *a*.

$$F(a) = \int_{-\infty}^{a} f(x)\mathrm{dx}$$



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Statistics as estimates of population characteristics

An estimate is based on a (random) sample from the population and is therefore not necessarily the correct value for the population. However, the commonly used estimates are all ***unbiased***, i.e. they do not differ from the true value in a systematic way.

Estimates may, on the other hand, differ from the true value in an unsystematic, ***random***, way. The purpose of a statistical analysis is, among other things, to ***quantify the uncertainty in the estimates***.

In a statistical analysis the random variation in the observations are used to derive a description of the random variation in the estimates based on a ***statistical model***, which is an idealized description of the processes that generate the data.

Using ***probability theory*** the uncertainty in the estimates can be deduced from the random variation in the sample.

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THE BINOMIAL DISTRIBUTION – A SIMPLE EXAMPLE OF A STATISTICAL MODEL

Consider a series of *n* experiments, where *n* is some fixed number.

Assume that
1. Each experiment has ***two possible outcomes***, usually referred to as “success” and “failure”.
2. The probability of a “success” is denoted *p* and is the ***same*** in all experiments.
3. The experiments are ***mutually independent*** i.e. knowledge of the outcome of one experiment does not change the probability of a “success” in another experiment.

These assumptions are e.g. fulfilled if we want to study coin tosses. Here we believe that *p*=0.5 and there is no need for an experimentally based estimate.

In other situations the probability of success is unknown and we may want to derive an estimate of this probability

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Intuitively the relative frequency of successes in the n experiments should be used to estimate the probability of a “success”.

What is the properties of this estimate?

If the ***binomial assumptions*** are fulfilled the following result is available:

The number of successes follows a ***binomial distribution***, i.e. the probability of getting ***exactly y successes in n experiments*** are given by the expression

$$P\{y \mid n, p\} = \binom{n}{y} p^y (1-p)^{n-y}$$

Here $p^y(1-p)^{n-y}$ represents the probability of a sequence if *y* successes *and n-y* failures and the binomial coefficient gives the number of such sequences of length *n.*

The binomial distribution with *n*=224 and *p*=0.5134 was used to derive percentiles for the number of boys per year born in Odder (page 5)

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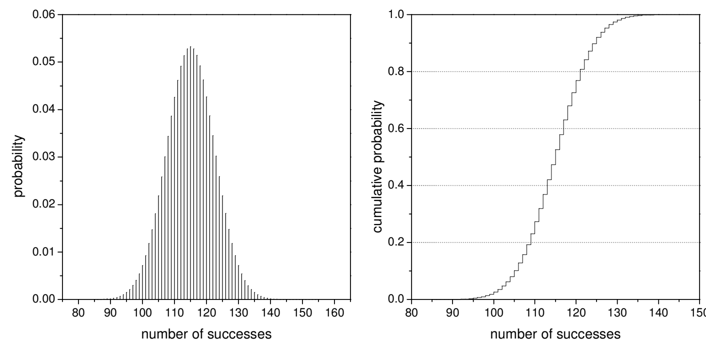
The ***probability function*** (left) and the ***cumulative probability function*** (right) for a binomial distribution with *n*=224 and *p*=0.5134



The ***expected number*** of successes, to be interpreted as the average number of successes in a large number of identical experiments, is

$$E(y) = \sum_{y=0}^{n} y \cdot P\{y \mid n, p\} = \cdots = n \cdot p$$

In the example above the expected value is therefore
$224 \cdot 0.5134 = 115.0$ boys out of 224 births.

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The ***variance*** of the number of successes, i.e. the expected value of the squared deviation from the mean, is

$$Var(y) = \sum_{y=0}^{n} \left(y - np\right)^2 P\{y \mid n, p\} = \cdots = n \cdot p(1-p)$$

and the ***standard deviation*** of the number of successes becomes

$$s.d.(y) = \sqrt{Var(y)} = \sqrt{np(1-p)}$$

In the example above we have

$$Var(y) = 224 \cdot 0.5134 \cdot (1 - 0.5134) = 55.96$$
$$s.d.(y) = \sqrt{Var(y)} = \sqrt{55.96} = 7.5$$

If the "experiment" was repeated a large number of times we would therefore expect the number of successes to fall in the interval from 100 to 130 approximately 95% of the times, since

$$mean \pm 2 \cdot s.d. = 115.0 \pm 2 \cdot 7.5 = 115 \pm 15$$

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The ***relative frequency*** (or proportion) is the number of successes divided by the number of experiments, viz.

$$\hat{p} = \frac{y}{n}$$

The distribution function of the relative frequency is therefore obtained from the distribution function of *y* by rescaling the *x*-axis (divide by *n*). Moreover

Expected value of the proportion

$$E(\hat{p}) = E\left(\frac{y}{n}\right) = \frac{n \cdot p}{n} = p$$

Variance and ***standard error*** of the proportion

$$Var(\hat{p}) = p(1-p)\big/n \qquad se(\hat{p}) = \sqrt{p(1-p)/n}$$

Interpretation: The expected value (or mean value), the variance and the standard error are the values of these quantities that one would find in a sample of proportions obtained by repeating the binomial experiment a large number of times and for each experiment compute the proportion.

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THE NORMAL DISTRIBUTION

The ***normal***, or ***Gaussian***, ***distribution*** is the most important theoretical distributions for continuous variables.

Normal distributions:

- A class of distributions of the same shape, but with different means and/or variances.
- Continuous distributions with sample space (the possible values) equal to all real numbers.

A normal distribution is completely determined by its ***mean*** and ***variance***. These are called the ***parameters*** of the distribution.

Notation: mean $= E(X) = \mu$, variance $= Var(X) = \sigma^2$, X ~ $N(\mu, \sigma^2)$

Density function : $f(x \mid \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$

There is no closed form expression for the ***cumulative distribution function***.

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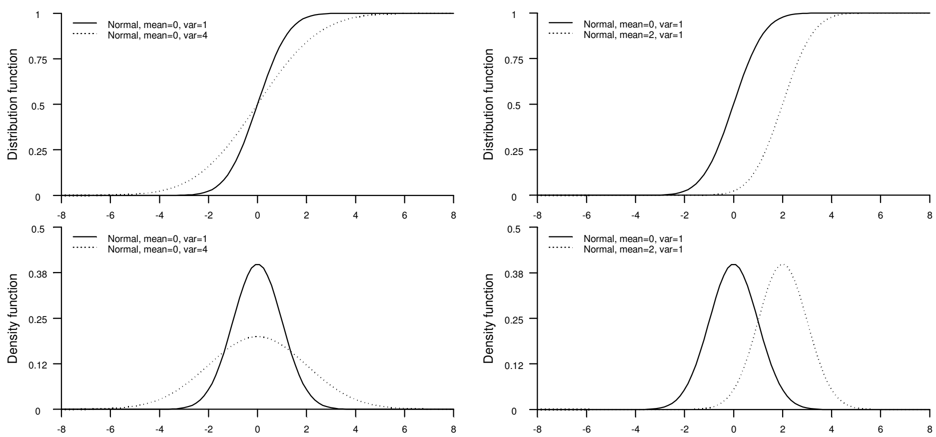
Examples of Normal distributions

same mean, but different variances   different means, same variance



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Relations between normal distributions

If X has a normal distribution with mean $E(X) = \mu$ and variance $Var(X) = \sigma^2$ then

$$Y = a + bX$$

has a normal distribution with mean $E(Y) = a + b\mu$ and variance $Var(y) = b^2\sigma^2$

The standardized variable

$$Z = \frac{X-\mu}{\sigma} = -\frac{\mu}{\sigma} + \frac{1}{\sigma}X$$

has a ***standard normal distribution***, i.e. a normal distribution with mean 0 and variance 1.

The cumulative distribution function and density function for a standard normal distribution are usually denoted Φ and φ, respectively. Tables of these function are widely available.

The density function of a normal distribution is symmetric about the mean.

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Selected values of the normal, cumulative distribution function

X	*P(X ≤ x)*
$\mu-3\sigma$	0.00135
$\mu-2\sigma$	0.02275
$\mu-\sigma$	0.1587
μ	0.5000
$\mu+\sigma$	0.8413
$\mu+2\sigma$	0.97725
$\mu+3\sigma$	0.99865

We see that

The probability of a value in the interval from *mean-sd* to *mean+sd* is approximately 68%

The probability of a value in the interval from *mean-2sd* to *mean+2sd* is approximately 95%

Exactly 95% of the values lies in the ***central prediction interval***:

$$\left[\mu-1.96\sigma, \mu+1.96\sigma\right]$$

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The use of the normal distribution in statistical analyses

Many frequency distributions resemble a normal probability distribution in shape, possibly after a suitable transformation of the data.

The name “normal” should not be taken literally to indicate that the distribution represents the normal behavior of random variation.

The importance of the normal distribution in statistics lies not so much in its ability to describe a wide range of observed frequency distributions, but in the central place it occupies in sampling theory.

In “large” samples the random error associated with parameter estimates and other statistics derived from the observations can usually be approximated very well by a normal distribution. This is a mathematical result which follows from the so-called central limit theorem.

Many statistical procedures assumes that the data can be considered as a random sample from a normal distribution. The adequacy of this assumption should always be assessed.

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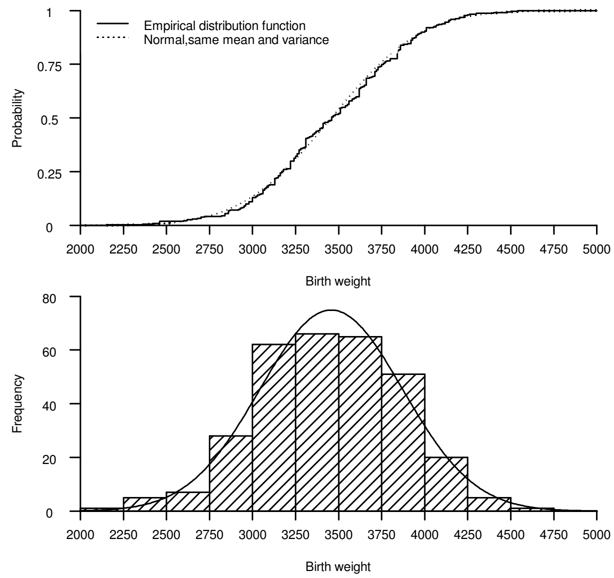
Example. Birth weight of 311 baby girls born in week 39



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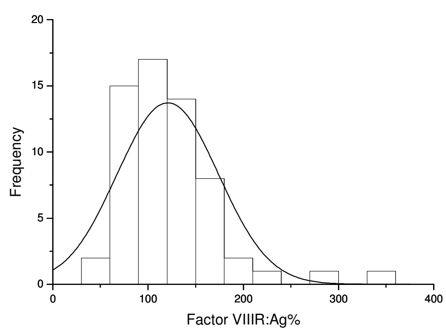
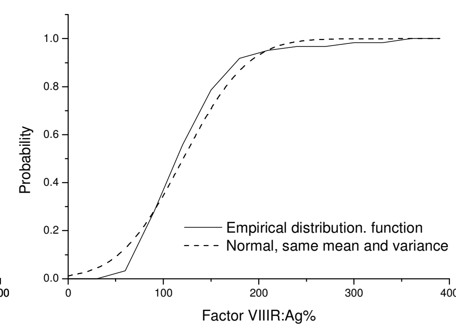
Example. Factor VIIIR:Ag % for 61 normal women





Comments:

The normal distribution gives a good description of the birth weight data, but is apparently not appropriate for the Factor VIIIR:Ag% data.

How do we decide if a normal distribution gives an adequate description?

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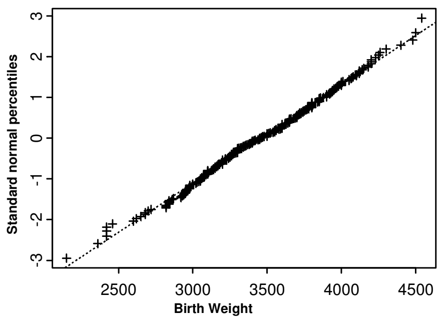
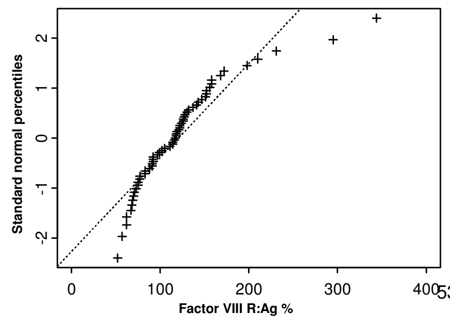
PROBABILITY PLOTTING (Q-Q PLOTS)

Problem: It is difficult to evaluate the goodness-of-fit of a normal distribution from histograms and plots of cumulative distribution functions.

Solution: A ***probability plot***, also called a ***Q-Q plot***. A plot of the cumulative distribution function in which the y-axis is transformed such that normal distributions becomes straight lines in the plot.

Y-axis: The probability is replaces by the corresponding standard normal percentile





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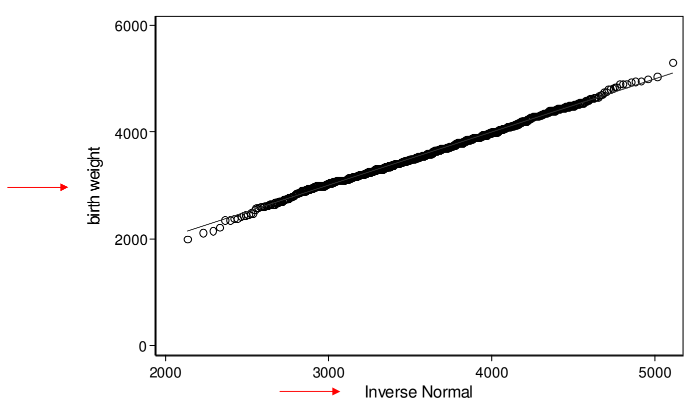
Q-Q plots with Stata

```
use skejby-cohort
qnorm bweight if year==1993 & gestage==40,mco(black)
```



Note: The axes are ***reversed*** compared to the usual Q-Q plots above. Also, Stata does ***not*** standardize the normal distribution

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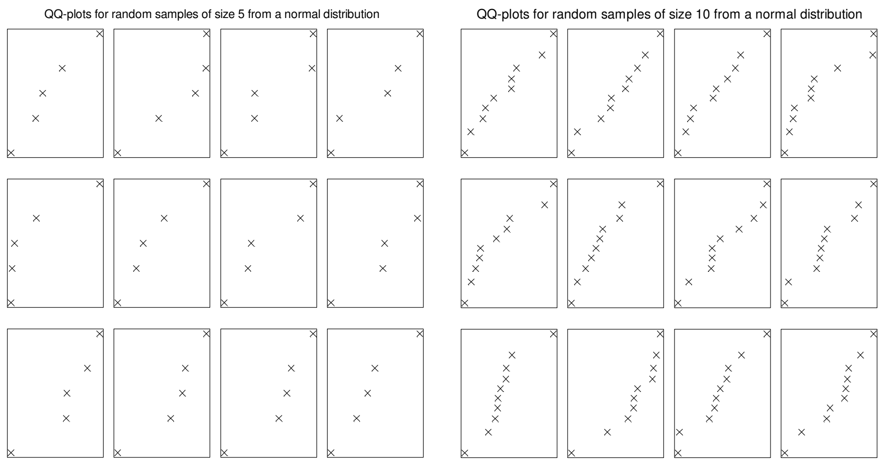
Examples of Q-Q plots of samples from normal populations 1



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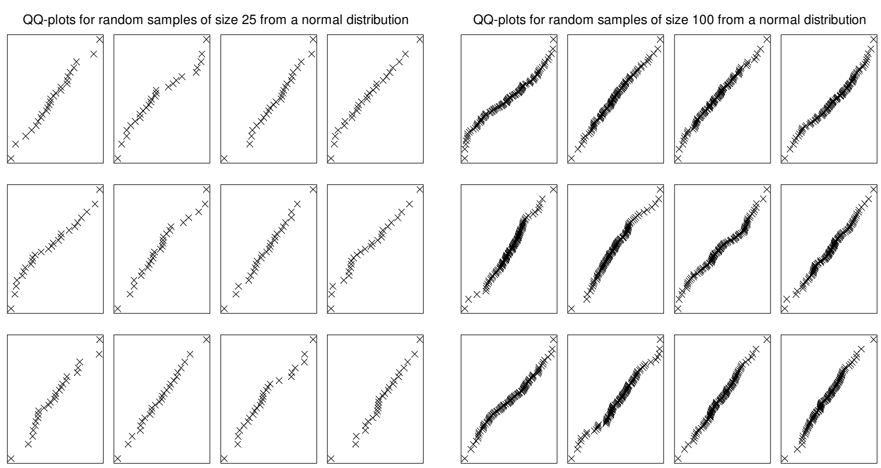
Examples of Q-Q plots of samples from normal populations 2



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