Age-specific incidence and prevalence

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Prevalence, incidence and duration: background model

Keiding (2005) Encycl.Biostat. (2. ed.), 2502-2506.

$$\xrightarrow{\beta} H \xrightarrow{\alpha} I$$

$$\mu \searrow \qquad \swarrow \nu$$

$$D$$

Individuals are born into the healthy state H according to time-homogeneous Poisson process with intensity β.

Individuals then develop according to (time, age, duration) - homogeneous Markov illness-death process:

Healthy individuals become diseased (I) with intensity α.
Healthy individuals die (D) with intensity μ.
Diseased individuals die with intensity ν.

Prevalence

Intuitively: the frequency of diseased in the population

Possible formalization: take a cross-sectional sample of the population at some time t, say $t = 0$.

$$E(\#\text{ healthy at } t=0) = \int_0^\infty \beta \exp\left[-(\alpha+\mu)a\right] da = \frac{\beta}{\alpha+\mu}$$

because individual born at $t = -a$ remains in H until t=0 with probability $\exp\left[-(\alpha+\mu)a\right]$.

$$E(\#\text{ diseased at } t=0) = \int_0^\infty \int_0^a \beta \exp\left[-(\alpha+\mu)y\right] \alpha \ \exp\left[-\nu(a-y)\right] dy\, da = \frac{\alpha\,\beta}{(\alpha+\mu)\nu}$$

$\left(\text{birth at } t = -a, \text{ disease onset at } t = -a+y \in [-a,0]\right)$.

Disease duration

Here exponentially distributed with mean

$$E(\text{disease duration}) = \nu^{-1}.$$

Incidence

Recommended ('individual') *definition* (R & G p. 32)

Healthy individual gets diseased at rate α per time unit.

Alternative definition,

Rate of occurrence of new disease in the population

$= E$ (new cases of diseased in population per calendar time unit)

$$= \beta \int_0^\infty \exp\left[-(\alpha+\mu)y\right] \alpha \, dy = \frac{\beta\alpha}{\alpha+\mu}$$

Prevalence = incidence × duration

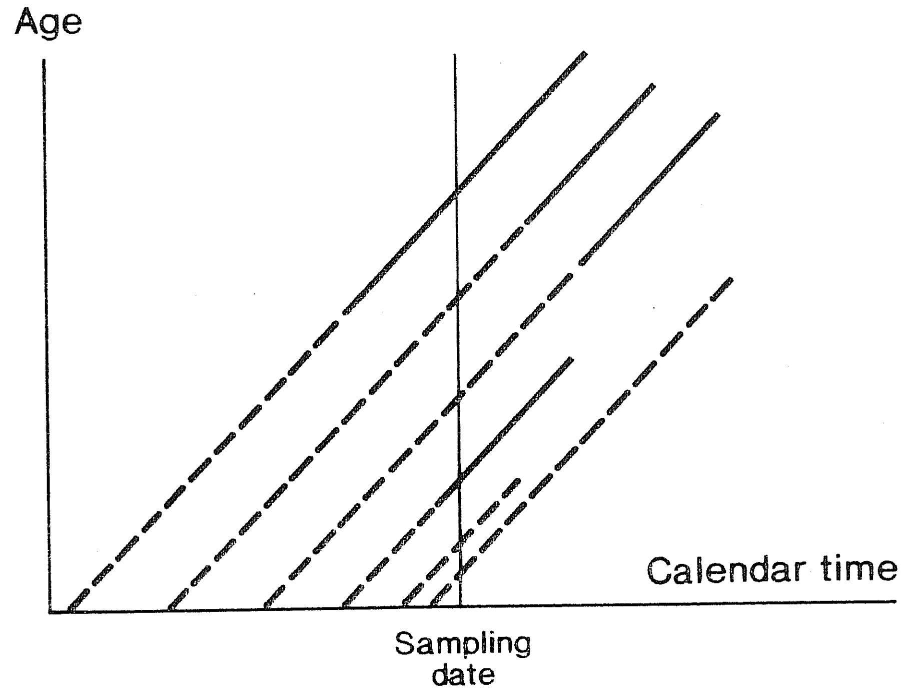
Incidence definition 1:

$$\text{prevalence odds} = \frac{E(\text{diseased})}{E(\text{healthy})} = \frac{\alpha\beta/\left[(\alpha+\mu)\nu\right]}{\beta/(\alpha+\mu)} = \alpha \cdot \frac{1}{\nu} = \text{incidence} \times \text{mean duration}$$

Incidence definition 2:

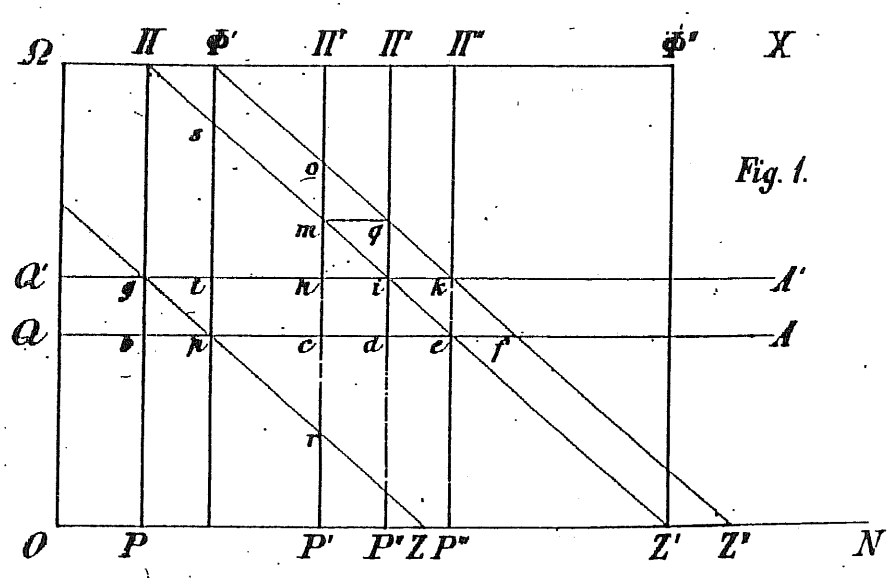
$$\text{prevalence} = E(\text{diseased}) = \frac{\alpha\beta}{(\alpha+\mu)\nu} = \frac{\alpha\beta}{\alpha+\mu} \cdot \frac{1}{\nu} = \text{incidence} \times \text{mean duration}$$

Age-dependence: Lexis diagram

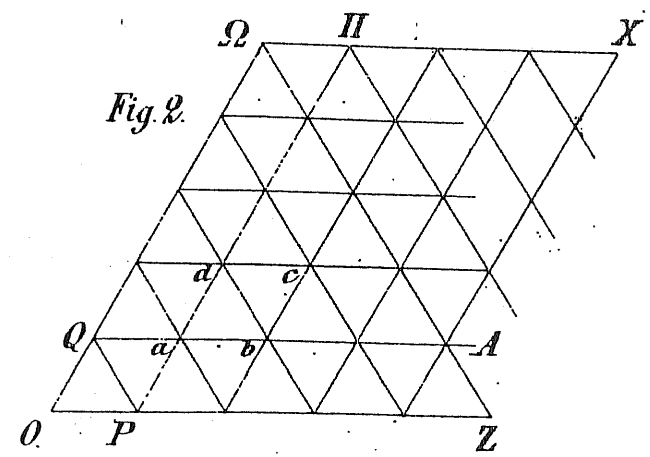


Lexis's own diagrams

Keiding (2000). Graphical representations in mortality measurement: Knapp, Zeuner, Becker, Lexis. Res. Rep. 00/8.

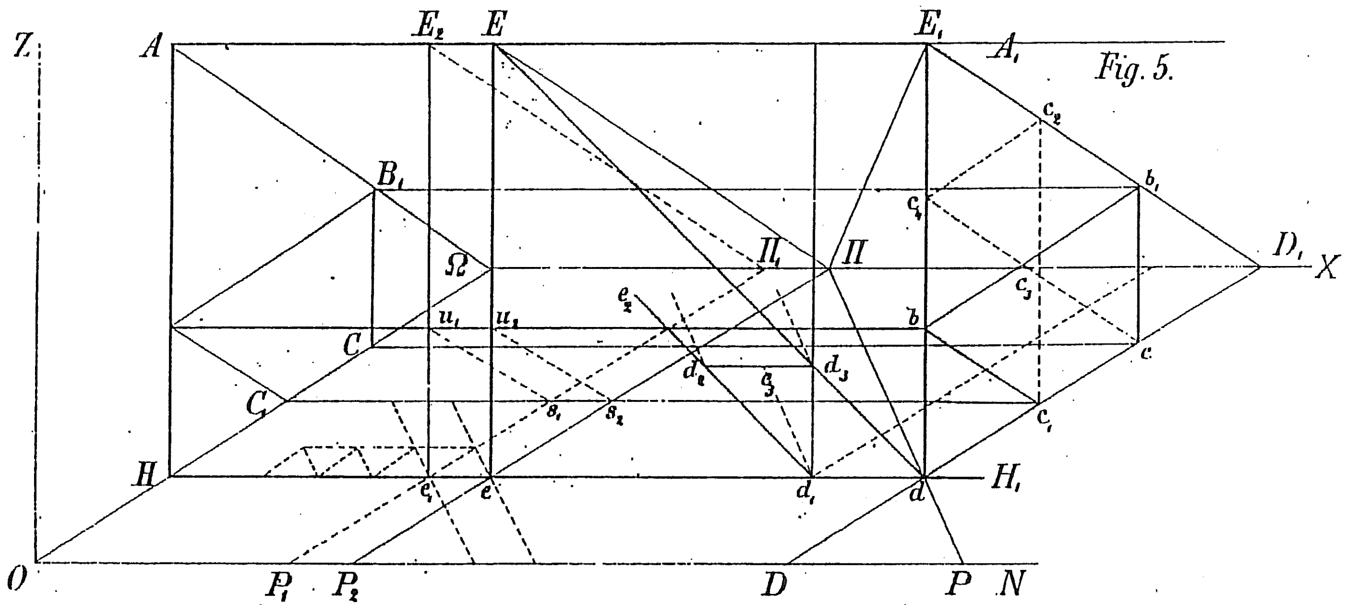


(Cohort, age)



equilateral

Lexis's own diagrams



(cohort, age at marriage, age at death)

Lexis diagram: basic concepts

Incidence $\alpha(t,a)$

Healthy person at time t and age a contracts disease in next interval $[t,t + h) \times [a,a + h)$ with probability $\alpha(t,a)\,h$

Mortality of the healthy $\mu(t,a)$

Healthy person at time t and age a dies in next interval $[t,t + h) \times [a,a + h)$ with probability $\mu(t,a)\,h$

Mortality of the diseased $\nu(t,a,d)$

Diseased person at time t and age a and duration d dies in next interval $[t,t + h) \times [a,a + h) \times [d,d + h)$ with probability $\nu(t,a,d)\,h$

Lexis diagram: the planar point process

Poisson process of births with intensity $\beta(t)$ generates

Planar Poisson process of incidences with intensity

$$\gamma(t,a)\alpha(t,a)$$

where

$$\gamma(t,a) = \beta(t-a)e^{-\int_0^a \left[\mu(t-a+y,y)+\alpha(t-a+y,y)\right]dy} \quad .$$

$\gamma(t,a)$: density of disease-free survivors

Brillinger (1986).

Prevalence under time homogeneity

$$\alpha(t,a) = \alpha(a); \qquad \mu(t,a) = \mu(a); \qquad \nu(t,a,d) = \nu(a,d)$$

$$E(\#\ \text{diseased at } t = 0)$$

$$= \int_0^\infty \int_0^\infty \beta \exp\left[-\int_0^y \{\alpha(u) + \mu(u)\} du\right] \alpha(y)\ \exp\left[-\int_0^s \nu(y+u,u)du\right] ds\ dy$$

where

$$\int_0^\infty \exp\left[-\int_0^s \nu(y+u,u)du\right] ds = E(\text{disease duration} \mid \text{onset at age } y) = \Delta_y$$

which is independent of y if lethality $\nu(a,d) = \nu(d)$ depends only on disease duration, not on age. In that case "prevalence"

$$= E(\#\ \text{diseased at } t = 0) = \Delta \int_0^\infty \beta \exp\left[-\int_0^y \{\alpha(u) + \mu(u)\} du\right] \alpha(y) dy$$

$$= \Delta \cdot E(\text{new cases of diseased in population per calendar time unit})$$

= mean duration · "incidence".