Solution of Exercise 1.3 (example)

1. 234 pairs have an average weight difference of 181 gram and a standard deviation of 534 gram. In one pair, the firstborn child is much larger than the second (-2440 gram). The dotplot shows a nearly normally distributed difference.

2. The 95% confidence interval for the average difference is (112g; 250g). As zero is not contained by the interval, there is a statistically significant difference in the weight of the two brothers. Our best estimate of the difference is that the younger brother weighs 181g more (cf. question 1), but the difference may be as small as 112g and up to 250g.

3. The 95% prediction interval is (-867g; 1228g). We would expect the central 95% of all the observations in the sample, as well as those in the underlying source population to lie in the interval such that 2,5% is below and 2,5% is above.

4. Since the data can reasonably be assumed to originate from a normal distribution (cf. the Q-Q-plot, straight line), have identical distribution, and are independent (each pair of brothers comes from different families), the hypothesis of no difference can be tested with a t-test. The resulting t_{obs} is 5.17, which in a t-distribution of 233 degrees of freedom yields a highly significant rejection of the null-hypothesis of no difference ($p < 0.001$). As the number of observations is rather large, this is virtually equivalent to an approximative z-test.